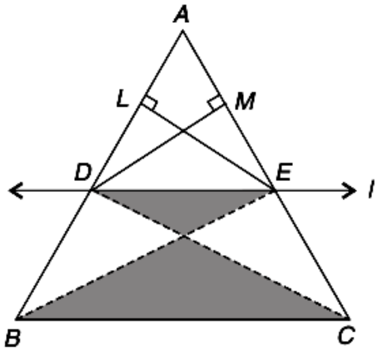
CHAPTER – 6
TRIANGLES

IMPORTANT THEOREMS

BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

GIVEN: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



TO PROVE : $$\frac{AD}{DB} = \frac{AE}{EC}$$

CONSTRUCTION : Join BE and CD. Draw $EL \perp$ to AB and $DM \perp AC$.

PROOF: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area } (\Delta BDE) = \text{area } (\Delta CDE) \quad \text{...(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \quad \text{...(ii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{...(iii)}$$

Put value form (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \quad \text{...(iv)}$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**

COROLLARY :

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ (ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$ (iv) $\frac{DB}{AB} = \frac{EC}{AC}$

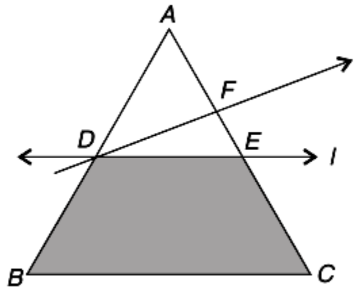
(v) $\frac{AD}{AB} = \frac{AE}{AC}$

CONVERSE OF BASIC PROPORTIONALITY THEOREM
(CONVERSE OF THALES THEOREM)

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

GIVEN : A Δ ABC and line 'l' intersecting the sides AB at D and AC at E such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



TO PROVE : $l \parallel BC$.

PROOF : Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{...(i) (Basic Proportionality Theorem)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{...(ii) (Given)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \quad \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow \quad FC = EC$$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$.

Hence, there is one and only line, $DE \parallel BC$, *i.e.*

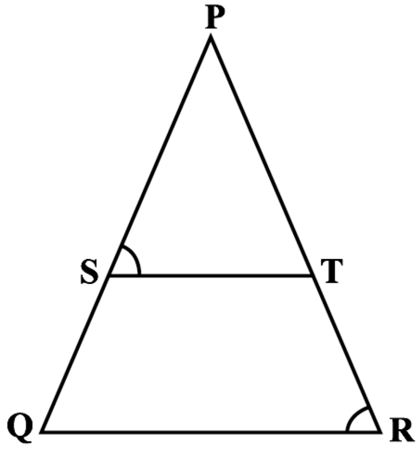
$$\boxed{l \parallel BC}$$

Hence Proved.

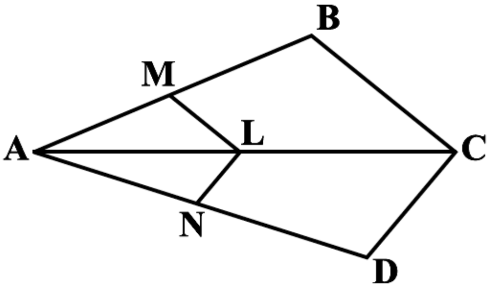
Exercise 6.2 Important Questions

1. If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC, prove that AD/AB = AE/AC
2. ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Show that AE/ED = BF/FC

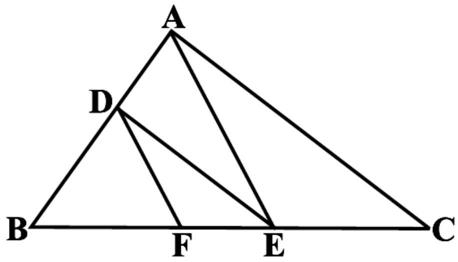
3. In the below figure, PS/SQ = PT/TR and ∠ PST = ∠ PRQ. Prove that PQR is an isosceles triangle.



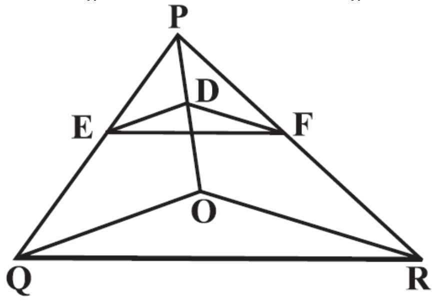
4. In the below figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD



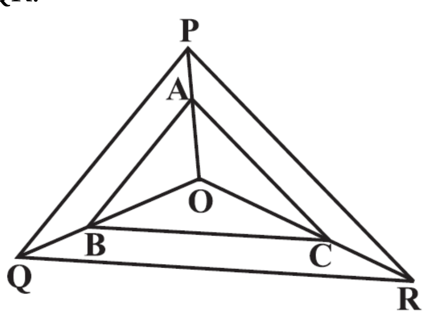
5. In the below figure, DE || AC and DF || AE. Prove that BF/FE = BE/EC



6. In the below figure, DE || OQ and DF || OR. Show that EF || QR.



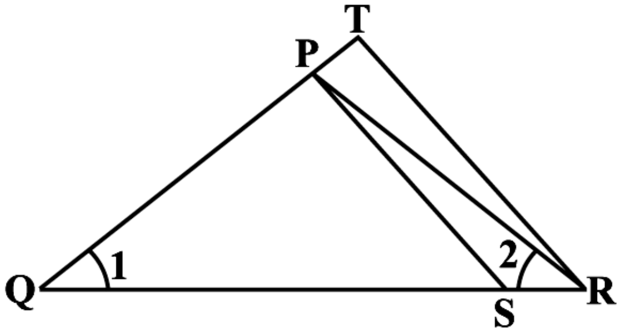
7. In the below figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



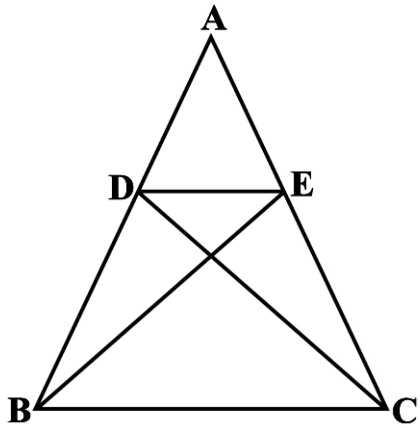
8. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO
9. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO· Show that ABCD is a trapezium.

Exercise 6.3 Important Questions

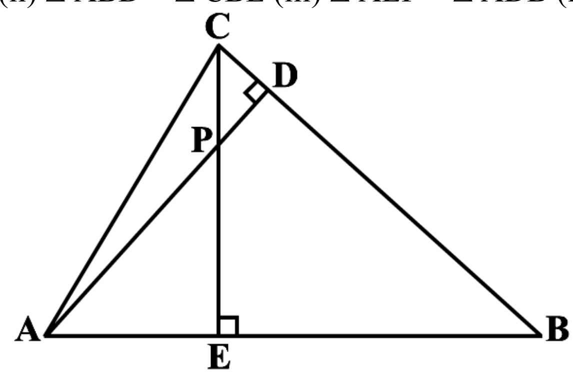
1. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.
2. In the below figure, QR/QS = QT/PR and ∠ 1 = ∠ 2. Show that Δ PQS ~ Δ TQR.



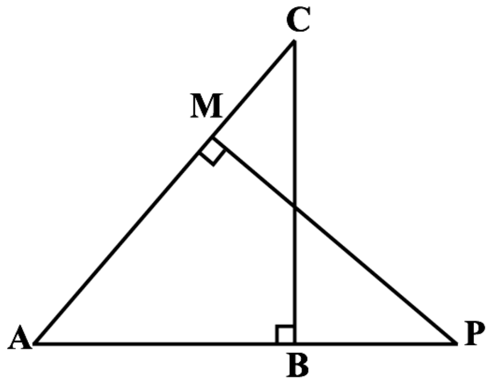
3. S and T are points on sides PR and QR of Δ PQR such that ∠ P = ∠ RTS. Show that Δ RPQ ~ Δ RTS.
4. In the below figure, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.



5. In the below figure, altitudes AD and CE of Δ ABC intersect each other at the point P. Show that: (i) Δ AEP ~ Δ CDP (ii) Δ ABD ~ Δ CBE (iii) Δ AEP ~ Δ ADB (iv) Δ PDC ~ Δ BEC



6. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.
7. In the below figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that: (i) Δ ABC ~ Δ AMP (ii) CA/PA = BC/MP



8. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR. Show that Δ ABC ~ Δ PQR.

9. D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that CA^2 = CB.CD.

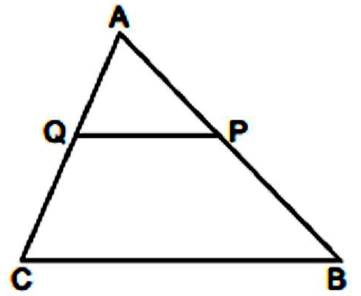
10. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

11. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

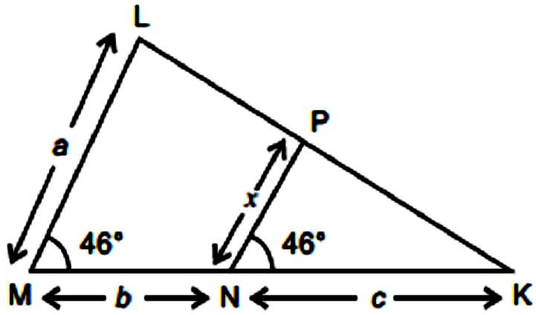
12. If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that AB/PQ = AD/PM·

IMPORTANT 1 MARK QUESTIONS

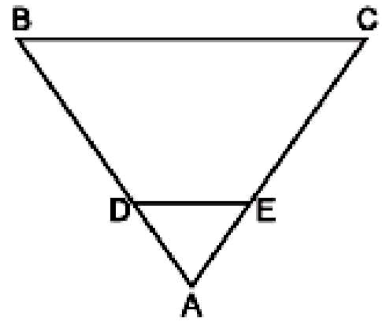
1. In ΔABC, D and E are points on sides AB and AC respectively such that DE || BC and AD : DB = 3 : 1. If EA = 6.6 cm then find AC.
2. In the fig., P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. I f PQ = 4.5 cm, find BC.



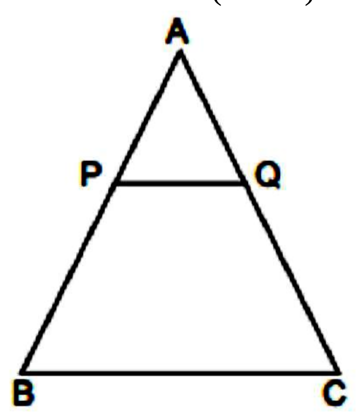
3. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB ?
4. In fig. ∠M = ∠N = 46°, express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



5. In figure, DE || BC in ΔABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.



6. In the fig., PQ || BC and AP : PB = 1 : 2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$

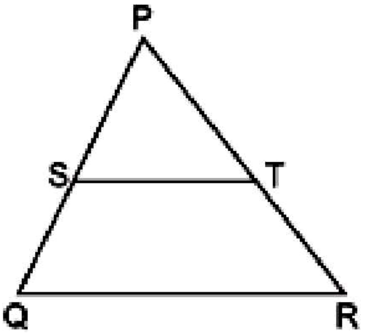


7. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Determine the height of the tower.

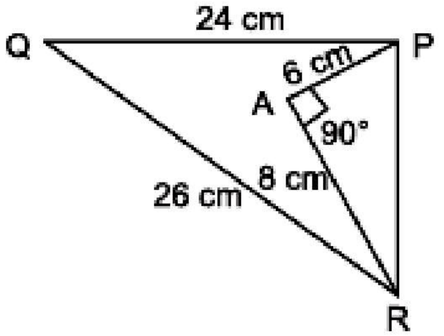
8. If ΔABC and ΔDEF are similar triangles such that ∠A = 57° and ∠E = 83°. Find C.

9. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

10. In figure, S and T are points on the sides PQ and PR, respectively of ΔPQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.

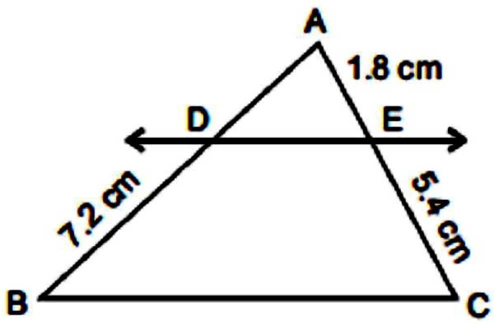


11. In the fig., PQ = 24 cm, QR = 26 cm, ∠PAR = 90°, PA = 6 cm and AR = 8 cm. Find ∠QPR.



12. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

13. In the given figure, DE || BC. Find AD.



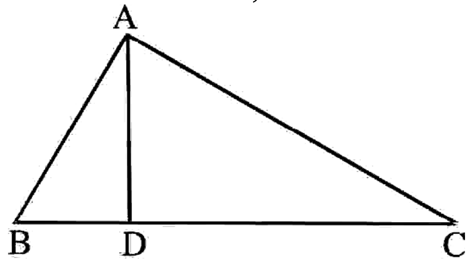
14. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm., what is the corresponding side of the other triangle ?

MCQ QUESTIONS (1 mark)

1. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is
(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

2. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?
(a) BC . EF = AC. FD (b) AB . EF = AC . DE (c) BC . DE = AB . EF (d) BC . DE = AB . FD

3. In the below, $\angle BAC = 90°$ and $AD \perp BC$. Then,



(a) $BD \cdot CD = BC^2$ (b) $AB \cdot AC = BC^2$ (c) $BD \cdot CD = AD^2$ (d) $AB \cdot AC = AD^2$

4. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

5. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are

(a) congruent but not similar (b) similar but not congruent

(c) neither congruent nor similar (d) congruent as well as similar

6. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true:

(a) DE = 12 cm, $\angle F = 50°$ (b) DE = 12 cm, $\angle F = 100°$

(c) EF = 12 cm, $\angle D = 100°$ (d) EF = 12 cm, $\angle D = 30°$

7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$

(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

8. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(ABC)}{ar(PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to

(a) 10 cm (b) 12 cm (c) 20/3 cm (d) 8 cm

9. If S is a point on side PQ of a ΔPQR such that PS = QS = RS, then

(a) $PR \cdot QR = RS^2$ (b) $QS^2 + RS^2 = QR^2$

(c) $PR^2 + QR^2 = PQ^2$ (d) $PS^2 + RS^2 = PR^2$

10. A vertical pole of length 20 m casts a shadow 10 m long on the ground and at the same time a tower casts a shadow 50 m long, then the height of the tower.

(a) 100 m (b) 120 m (c) 25 m (d) none of these

11. The areas of two similar triangles are in the ratio 4 : 9. The corresponding sides of these triangles are in the ratio

(a) 2 : 3 (b) 4 : 9 (c) 81 : 16 (d) 16 : 81

12. The areas of two similar triangles are in respectively 9 cm^2 and 16 cm^2. The ratio of their corresponding sides is

(a) 2 : 3 (b) 3 : 4 (c) 4 : 3 (d) 4 : 5

13. Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. The ratio of their corresponding heights is
(a) 3 : 2 (b) 5 : 4 (c) 5 : 7 (d) 4 : 5

14. If ΔABC and ΔDEF are similar such that 2AB = DE and BC = 8 cm, then EF =
(a) 16 cm (b) 112 cm (c) 8 cm (d) 4 cm

15. XY is drawn parallel to the base BC of a ΔABC cutting AB at X and AC at Y. If AB = 4BX and YC = 2 cm, then AY =
(a) 2 cm (b) 6 cm (c) 8 cm (d) 4 cm

16. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, the distance between their tops is
(a) 14 cm (b) 12 cm (c) 13 cm (d) 11 cm

17. In triangles ABC and DEF, $\angle A = \angle E = 40^0$, AB : ED = AC : EF and $\angle F = 65^0$, then $\angle B =$
(a) 35^0 (b) 65^0 (c) 75^0 (d) 85^0

18. If ABC and DEF are similar triangles such that $\angle A = 47^0$ and $\angle E = 83^0$, then $\angle C=$
(a) 50^0 (b) 60^0 (c) 70^0 (d) 80^0