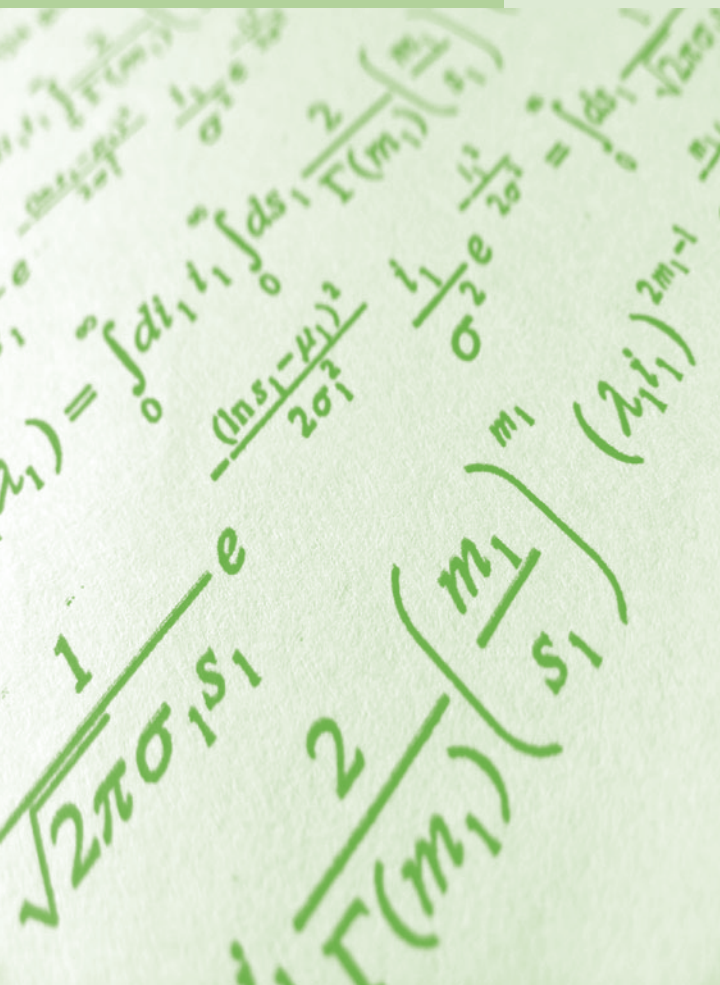


# Chapter 1

# Number Systems



## REMEMBER

Before beginning this chapter, you should be able to:

- Review the types of numbers and understand representation of numbers on a number line
- Study properties of numbers

## KEY IDEAS

After completing this chapter, you would be able to:

- Study Euclid's division lemma
- Learn fundamental theorem of arithmetic
- Prove theorems related to irrational numbers and rational numbers
- Study proof of irrationality

## INTRODUCTION

Earlier we have learnt about number line, irrational numbers, how to represent irrational numbers on the number line, real numbers and their decimal representation, representing real numbers on the number line and operations on the number line. We shall now present a detailed study of Euclid's division algorithm and fundamental theorem of arithmetic. Further we continue the revision of irrational numbers and the decimal expansion of rational numbers.

We are very familiar with division rule.

That is,  $\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}$ .

**Euclid's division lemma** is based on this rule. We use this result to obtain the HCF of two numbers. And also we know that, every composite number can be expressed as the product of primes in a unique way. This is the fundamental theorem of arithmetic. This result is used to prove the irrationality of a number.

In the previous class we have studied about rational numbers. In this chapter, we shall explore when exactly the decimal expansion of a rational number say  $\frac{p}{q}$  ( $q \neq 0$ ), is terminating and when it is non-terminating repeating.

## EUCLID'S DIVISION LEMMA

For any two positive integers, say  $x$  and  $y$ , there exist unique integers say  $q$  and  $r$  satisfying  $x = yq + r$  where  $0 \leq r < y$ .

**Example:** Consider the integers 9 and 19.

$$19 = 9 \times 2 + 1$$

**Example:** Consider the integers 6 and 24.

$$24 = 6 \times 4 + 0$$

### Notes

1. Euclid's division algorithm is used for finding the greatest common divisor of two numbers.
2. An algorithm is a process of solving particular problems.

### EXAMPLE 1.1

Find the HCF of 250 and 30.

#### SOLUTION

By using Euclid's division lemma, we get,

$$250 = 30 \times 8 + 10$$

Now consider, the divisor and remainder.

Again by using Euclid's division lemma, we get

$$30 = 10 \times 3 + 0$$

Here, we notice the remainder is zero and we cannot proceed further.

The divisor at this stage is 10.

The HCF of 250 and 30 is 10.

It can be verified by listing out all the factors of 250 and 30.

**Note** Euclid's Division Algorithm is stated for only positive integers, it can be extended for all negative integers.

Euclid's Division Algorithm has several applications. The following examples give the idea of the applications.

### EXAMPLE 1.2

Show that every positive even integer is of the form  $2n$  and every positive odd integer is of the form  $2n + 1$ .

#### SOLUTION

For any integer  $x$  and  $y = 2$ ,  $x = 2n + r$ , where  $n \geq 0$

But  $0 \leq r < 2$

$$\Rightarrow r = 0 \text{ or } 1$$

When  $r = 0$ ,  $x = 2n$

$$\Rightarrow x \text{ is a positive even integer}$$

When  $r = 1$ ,  $x = 2n + 1$

$$\Rightarrow x \text{ is a positive odd integer.}$$

### EXAMPLE 1.3

A trader has 612 Dettol soaps and 342 Pears soaps. He packs them in boxes and each box contains exactly one type of soap. If every box contains the same number of soaps, then find the number of soaps in each box such that the number of boxes is the least.

#### SOLUTION

The required number is HCF of 612 and 342.

This number gives the maximum number of soaps in each box and the number of boxes with them be the least.

By using Euclid's division algorithm, we have

$$612 = 342 \times 1 + 270$$

$$342 = 270 \times 1 + 72$$

$$270 = 72 \times 3 + 54$$

$$72 = 54 \times 1 + 18$$

$$54 = 18 \times 3 + 0$$

Here we notice that the remainder is zero, and the divisor at this stage is 18.

$\therefore$  HCF of 612 and 342 is 18.

So, the trader can pack 18 soaps per box.

## FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be expressed as the product of prime factors uniquely.

**Note** In general  $a = p_1 p_2 p_3, \dots, p_n$  where  $p_1, p_2, p_3, \dots, p_n$  are primes in ascending order.

**EXAMPLE 1.4**

Write 1800 as product of prime factors.

**SOLUTION**

$$\begin{array}{r}
 2 \overline{)1800} \\
 \underline{2 \phantom{00}900} \\
 2 \overline{)450} \\
 \underline{2 \phantom{00}225} \\
 3 \overline{)225} \\
 \underline{3 \phantom{00}75} \\
 3 \overline{)75} \\
 \underline{5 \phantom{00}25} \\
 5 \overline{)25} \\
 \underline{5 \phantom{00}0}
 \end{array}$$

$$\therefore 1800 = 2^3 \times 3^2 \times 5^2$$

Let us see the applications of fundamental theorem.

**EXAMPLE 1.5**

Check whether there is any value of  $x$  for which  $6^x$  ends with 5.

**SOLUTION**

If  $6^x$  ends with 5, then  $6^x$  would contain the prime number 5.

$$\text{But, } 6^x = (2 \times 3)^x = 2^x \times 3^x$$

$\Rightarrow$  The prime numbers in the factorization of  $6^x$  are 2 and 3

By uniqueness of fundamental theorem, there are no prime numbers other than 2 and 3 in  $6^x$ .

$\therefore 6^x$  never ends with 5.

**EXAMPLE 1.6**

Show that  $5 \times 3 \times 2 + 3$  is a composite number.

**SOLUTION**

$$5 \times 3 \times 2 + 3 = 3(5 \times 2 + 1) = 3(11) = 3 \times 11$$

$\therefore$  The given number is a composite number.

**EXAMPLE 1.7**

Find the HCF and LCM of 48 and 56 by prime factorization method.

**SOLUTION**

$$48 = 2^4 \times 3^1$$

$$56 = 2^3 \times 7^1$$

HCF =  $2^3$  (The product of common prime factors with lesser index)

LCM =  $2^4 \times 3^1 \times 7^1$  (product of common prime factors with greater index).

### EXAMPLE 1.8

Find the HCF and LCM of 36, 48 and 60 by prime factorization method.

#### SOLUTION

$$36 = 2^2 \times 3^2$$

$$48 = 2^4 \times 3^1$$

$$60 = 2^2 \times 3^1 \times 5^1$$

$$\text{HCF} = 2^2 \times 3^1 = 12$$

$$\text{LCM} = 2^4 \times 3^2 \times 5^1 = 720.$$

### EXAMPLE 1.9

Two bells toll at intervals of 24 minutes and 36 minutes respectively. If they toll together at 9 am, after how many minutes do they toll together again, at the earliest?

#### SOLUTION

The required time is LCM of 24 and 36.

$$24 = 2^3 \times 3^1$$

$$36 = 2^2 \times 3^2$$

$$\therefore \text{LCM of 24 and 36 is } 2^3 \times 3^2 = 72.$$

So, they will toll together after 72 minutes.

### EXAMPLE 1.10

There are 44 boys and 32 girls in a class. These students are arranged in rows for a prayer in such a way that each row consists of only either boys or girls, and every row contains an equal number of students. From the following 4 options, choose the minimum number of rows in which all the students can be arranged.

(a) 4

(b) 12

(c) 15

(d) 19

#### SOLUTION

For number of rows to be the least, the number of students in each row must be the greatest. The number of students in each row is a common factor of 44 and 32. The greatest value is the HCF of 44 and 32, which is 4.

$$\therefore \text{Minimum number of rows} = \frac{44}{4} + \frac{32}{4} = 19.$$

### EXAMPLE 1.11

Two positive numbers have their HCF as 12 and their product as 6336. Choose the correct option for the number of pairs possible for the given numbers.

(a) 2

(b) 3

(c) 4

(d) 5

### SOLUTION

Let the numbers be  $12x$  and  $12y$ , where  $x$  and  $y$  are co-primes.

Product of the numbers =  $144xy$

$$144xy = 6336$$

$$xy = 44$$

44 can be written as the product of two factors in three ways, i.e.,  $1 \times 44$ ,  $2 \times 22$ ,  $4 \times 11$ .

As  $x$  and  $y$  are relatively prime,  $(x, y)$  can be  $(1, 44)$  or  $(4, 11)$  but not  $(2, 22)$ .

Hence two possible pairs exist.

### EXAMPLE 1.12

Four persons P, Q, R and S start running around a circular track simultaneously. If they complete one round in 10, 8, 12 and 18 minutes respectively, after how much time will they next meet at the starting point? Choose the right answer from the given options.

(a) 180 minutes    (b) 270 minutes    (c) 360 minutes    (d) 450 minutes

### SOLUTION

The time interval for them to meet at the starting point for the next time = LCM (10, 8, 12, 18) = 360 minutes.

## IRRATIONAL NUMBERS

A number which cannot be written in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers and  $q \neq 0$ .

**Example:**  $\sqrt{2}, \sqrt{3}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, 0.123421635, \dots$ , etc.

### Theorem 1

If  $p$  divides  $x^3$ , then  $p$  divides  $x$ , where  $x$  is a positive integer and  $p$  is a prime number.

**Proof:** Let  $x = p_1 p_2 \dots p_n$ , where  $p_1, p_2, p_3, \dots, p_n$  are primes, not necessarily distinct.

$$\Rightarrow x^3 = p_1^3 p_2^3 \dots p_n^3$$

Given that  $p$  divides  $x^3$ .

By fundamental theorem,  $p$  is one of the primes of  $x^3$ .

By the uniqueness of fundamental theorem, the distinct primes of  $x^3$  are same as the distinct primes of  $x$ .

$\Rightarrow p$  divides  $x$

Hence proved.

Similarly, if  $p$  divides  $x^2$  then  $p$  divides  $x$ , where  $p$  is a prime number and  $x$  is a positive integer.

## Theorem 2

Prove that  $\sqrt{2}$  is irrational.

**Proof:** Let us assume that,  $\sqrt{2}$  is not irrational.

So,  $\sqrt{2}$  is rational.

$$\Rightarrow \sqrt{2} = \frac{x}{y}, \text{ where } x, y \text{ are integers and } y \neq 0$$

Let  $x$  and  $y$  be co-primes.

Taking squares on both sides,

$$\begin{aligned} \Rightarrow 2 &= \frac{x^2}{y^2} \\ \Rightarrow 2y^2 &= x^2 \\ \Rightarrow 2 &\text{ divides } x^2 \\ \Rightarrow 2 &\text{ divides } x \end{aligned} \tag{1}$$

$\therefore$  For some integer  $z$ ,

$$x = 2z \tag{2}$$

From Eqs. (1) and (2),

$$\begin{aligned} 2y^2 &= 4z^2 \\ \Rightarrow y^2 &= 2z^2 \\ \Rightarrow 2 &\text{ divides } y^2 \\ \Rightarrow 2 &\text{ divides } y \end{aligned}$$

$\therefore x$  and  $y$  have at least 2 as a common factor.

But it contradicts the fact that  $x$  and  $y$  are co-primes.

$\therefore \sqrt{2}$  is irrational.

### EXAMPLE 1.13

Prove that  $\sqrt{5}$  is irrational.

#### SOLUTION

Let us assume that  $\sqrt{5}$  is not irrational.

$\therefore \sqrt{5}$  is rational.

$$\Rightarrow \sqrt{5} = \frac{p}{q}$$

Let  $p, q$  be co-primes.

Taking squares on both the sides,

$$\begin{aligned} \Rightarrow 5 &= \frac{p^2}{q^2} \\ \Rightarrow p^2 &= 5q^2 \end{aligned} \tag{1}$$

$$\Rightarrow 5 \text{ divides } p^2$$

$$\Rightarrow 5 \text{ divides } p$$

$\therefore$  For some integer  $r$ ,

$$p = 5r \tag{2}$$

From Eqs. (1) and (2),

$$25r^2 = 5q^2$$

$$\Rightarrow q^2 = 5r^2$$

$$\Rightarrow 5 \text{ divides } q^2$$

$$\Rightarrow 5 \text{ divides } q$$

$\therefore p$  and  $q$  have at least 5 as a common factor.

But it contradicts the fact that  $p$  and  $q$  are co-primes.

$\therefore \sqrt{5}$  is irrational.

### EXAMPLE 1.14

Show that  $3 + \sqrt{2}$  is irrational.

#### SOLUTION

Let us assume that  $3 + \sqrt{2}$  is rational.

$\therefore 3 + \sqrt{2} = \frac{p}{q}$ , where  $p$  and  $q$  are integers.

$$\Rightarrow \sqrt{2} = \frac{p}{q} - 3 \Rightarrow \sqrt{2} = \frac{p - 3q}{q}$$

Since  $p$  and  $q$  are integers,  $\frac{p - 3q}{q}$  is rational.

But  $\sqrt{2}$  is irrational.

It contradicts our assumption that  $3 + \sqrt{2}$  is rational.

$\therefore$  Our assumption is wrong.

Hence,  $3 + \sqrt{2}$  is irrational.

### EXAMPLE 1.15

If  $a = \sqrt{11} + \sqrt{3}$ ,  $b = \sqrt{12} + \sqrt{2}$  and  $c = \sqrt{6} + \sqrt{4}$ , then which of the following holds true?

(a)  $c > a > b$

(b)  $a > b > c$

(c)  $a > c > b$

(d)  $b > a > c$

#### SOLUTION

$$a = \sqrt{11} + \sqrt{3}$$

$$a^2 = (\sqrt{11} + \sqrt{3})^2 = 14 + 2\sqrt{33}$$

$$b = \sqrt{12} + \sqrt{2}$$

$$b^2 = (\sqrt{12} + \sqrt{2})^2 = 14 + 2\sqrt{24}$$

As  $\sqrt{33} > \sqrt{24}$ ,  $a^2 > b^2$ , hence  $a > b$ .

$$c = \sqrt{6} + \sqrt{4}$$

$$c^2 = (\sqrt{6} + \sqrt{4})^2 = 10 + 2\sqrt{24}$$

As  $14 > 10$ ,  $b^2 > c^2$ , hence  $b > c$ .

Hence,  $a > b > c$ .

### EXAMPLE 1.16

Choose the correct value of  $\frac{1}{\sqrt{9} + \sqrt{10}} + \frac{1}{\sqrt{10} + \sqrt{11}} + \frac{1}{\sqrt{11} + \sqrt{12}} + \dots$  up to 91 terms from the following options.

(a) 7

(b) 8

(c) 6

(d) 9

### SOLUTION

$$\frac{1}{\sqrt{9} + \sqrt{10}} = \frac{\sqrt{9} - \sqrt{10}}{(\sqrt{9} - \sqrt{10})(\sqrt{9} - \sqrt{10})}$$

(multiplying both its numerator and denominator by  $\sqrt{9} - \sqrt{10}$ )

$$\begin{aligned} \text{Hence, } \frac{1}{\sqrt{9} + \sqrt{10}} &= \frac{\sqrt{9} - \sqrt{10}}{(\sqrt{9})^2 - (\sqrt{10})^2} \\ &= \frac{\sqrt{9} - \sqrt{10}}{-1} \end{aligned}$$

By multiplying numerator and denominator of each term by the conjugate of their denominators, the given expression becomes

$$\begin{aligned} &\frac{\sqrt{9} - \sqrt{10}}{-1} + \frac{\sqrt{10} - \sqrt{11}}{-1} + \frac{\sqrt{11} - \sqrt{12}}{-1} + \dots + \frac{\sqrt{99} - \sqrt{100}}{-1} \\ &= \frac{\sqrt{9} - \sqrt{100}}{-1} = 7. \end{aligned}$$

## RATIONAL NUMBERS

Numbers which can be written in the form of  $\frac{p}{q}$  ( $q \neq 0$ ), where  $p, q$  are integers, are called rational numbers.

**Note** Every terminating decimal and non-terminating repeating decimal can be expressed in the form  $\frac{p}{q}$  ( $q \neq 0$ ).

**Examples:**

1.  $0.27 = \frac{27}{100} = \frac{27}{2^2 \times 5^2}$

2.  $2.356 = \frac{2356}{1000} = \frac{2^2 \times 589}{2^3 \times 5^3} = \frac{589}{2 \times 5^3}$

3.  $2.0325 = \frac{20325}{10000} = \frac{3 \times 5^2 \times 271}{2^4 \times 5^4} = \frac{3 \times 271}{2^4 \times 5^2}$

From the above examples, we notice that every terminating decimal can be written in the form of  $\frac{p}{q}$  ( $q \neq 0$ ), where  $p$  and  $q$  are co-primes and  $q$  is of the form  $2^m \cdot 5^n$  ( $m$  and  $n$  are non-negative integers).

Let us write this result formally.

### Theorem 3

Let  $a$  be a terminating decimal. Then  $a$  can be expressed as  $\frac{p}{q}$  ( $q \neq 0$ ), where  $p$  and  $q$  are co-primes, and the prime factorization of  $q$  is of the form  $2^m \cdot 5^n$ .

Let us observe the following examples:

1.  $\frac{1}{4} = \frac{1}{2^2} = \frac{1 \times 5^2}{2^2 \times 5^2} = \frac{2}{25} = 0.25$

2.  $\frac{7}{25} = \frac{7}{5^2} = \frac{7 \times 2^2}{5^2 \times 2^2} = \frac{28}{100} = 0.28$

3.  $\frac{23}{125} = \frac{23}{5^3} = \frac{23 \times 2^3}{5^3 \times 2^3} = \frac{184}{1000} = 0.184$

4.  $\frac{147}{50} = \frac{147}{2 \times 5^2} = \frac{147 \times 2}{2^2 \times 5^2} = \frac{294}{100} = 2.94$

From the above examples, we notice that every rational number of the form  $\frac{p}{q}$  ( $q \neq 0$ ), where  $q$  is of the form  $2^m \cdot 5^n$  can be written as  $\frac{x}{y}$ , where  $y$  is of the form  $10^k$ ,  $k$  being a natural number.

Let us write this result formally.

### Theorem 4

If  $\frac{p}{q}$  is a rational number where  $q$  is of the form  $2^m \cdot 5^n$  ( $m \in W$ ), then  $\frac{p}{q}$  has a terminating decimal expansion.

Let us observe the following examples.

**Examples:**

1.  $\frac{5}{3} = 1.66666... = 1.\overline{6}$

2.  $\frac{8}{7} = 1.142857$

3.  $\frac{1}{11} = 0.09$

### Theorem 5

If  $\frac{p}{q}$  is a rational number and  $q$  is not of the form  $2^m \cdot 5^n$  ( $m$  and  $n \in \mathbb{W}$ ), then  $\frac{p}{q}$  has a non-terminating repeating decimal expansion.

#### EXAMPLE 1.17

Which of the following rational numbers are terminating decimals?

(a)  $\frac{17}{2^3 \times 5^2}$

(b)  $\frac{25}{3^2 \times 2^3}$

(c)  $\frac{68}{2^2 \times 5^2 \times 7^2}$

(d)  $\frac{125}{3^3 \times 7^2}$

#### SOLUTION

Clearly  $\frac{17}{2^3 \times 5^2}$  is the only terminating decimal and the remaining are non-terminating decimals.

## TEST YOUR CONCEPTS

### Very Short Answer Type Questions

- $\frac{7^3}{5^4}$  is a non-terminating repeating decimal. (True/False)
- $0.\bar{5} - 0.\overline{49} =$  \_\_\_\_\_.
- If  $360 = 2^x \times 3^y \times 5^z$ , then  $x + y + z =$  \_\_\_\_\_.
- If  $a = 2 + \sqrt{3}$  and  $b = \sqrt{2} - \sqrt{3}$ , then  $a + b$  is \_\_\_\_\_. (rational/irrational)
- If 19 divides  $a^3$  (where  $a$  is a positive integer), then 19 divides  $a$ . (True/False)
- If  $0.\bar{7} = \frac{p}{q}$ , then  $p + q =$  \_\_\_\_\_.
- Product of two irrational numbers is an irrational number. (True/False)
- $2 \times 3 \times 15 + 7$  is a \_\_\_\_\_. (prime number/composite number)
- Product of LCM and HCF of 25 and 625 is \_\_\_\_\_.
- For what values of  $x$ ,  $2^x \times 5^x$  ends in 5.

### Short Answer Type Questions

- Find the HCF of 72 and 264 by using Euclid's division algorithm.
- Show that  $7 \times 5 \times 3 \times 2 + 7$  is a composite number.
- Find the HCF and LCM of 108 and 360 by prime factorization method.
- Prove that  $\sqrt{6}$  is irrational.
- Without performing the actual division, check whether the following rational numbers are terminating or non-terminating.  
(i)  $\frac{23}{175}$  (ii)  $\frac{125}{325}$  (iii)  $\frac{73}{40}$  (iv)  $\frac{157}{125}$

### Essay Type Questions

- A fruit vendor has 732 apples and 942 oranges. He distributes these fruits among the students of an orphanage, such that each of them gets either only apples or only oranges in equal number. Find the least possible number of students.
- Write 75600 as the product of prime factors.
- Aloukya and Manoghna run around a circular track and they take 180 seconds and 150 seconds respectively to complete one revolution. If they start together at 9 am from the same point, how long it would take for them to meet again for the first time at the starting point?
- Prove that  $5 - \sqrt{5}$  is irrational.
- Express  $23.\overline{324}$  in  $\frac{p}{q}$  form, where  $p$  and  $q$  are integers.

## CONCEPT APPLICATION

### Level 1

- If  $n$  is a natural number, then  $9^{2n} - 4^{2n}$  is always divisible by \_\_\_\_\_.  
(a) 5  
(b) 13  
(c) Both (a) and (b)  
(d) Neither (a) nor (b)
- $N$  is a natural number such that when  $N^3$  is divided by 9, it leaves remainder  $a$ . It can be concluded that  
(a)  $a$  is a perfect square.  
(b)  $a$  is a perfect cube.  
(c) Both (a) and (b)  
(d) Neither (a) nor (b)



3. The remainder of any perfect square divided by 3 is \_\_\_\_\_.  
 (a) 0  
 (b) 1  
 (c) Either (a) or (b)  
 (d) Neither (a) nor (b)
4. Find the HCF of 432 and 504 using prime factorization method.  
 (a) 36 (b) 72  
 (c) 96 (d) 108
5. If  $n$  is any natural number, then  $6^n - 5^n$  always ends with \_\_\_\_\_.  
 (a) 1 (b) 3  
 (c) 5 (d) 7
6. The LCM of two numbers is 1200. Which of the following cannot be their HCF?  
 (a) 600 (b) 500  
 (c) 200 (d) 400
7. Which of the following is always true?  
 (a) The rationalizing factor of a number is unique.  
 (b) The sum of two distinct irrational numbers is rational.  
 (c) The product of two distinct irrational numbers is irrational.  
 (d) None of these
8. Find the remainder when the square of any number is divided by 4.  
 (a) 0 (b) 1  
 (c) Either (a) or (b) (d) Neither (a) nor (b)
9. Ashok has two vessels which contain 720 ml and 405 ml of milk respectively. Milk in each vessel is poured into glasses of equal capacity to their brim. Find the minimum number of glasses which can be filled with milk.  
 (a) 45 (b) 35  
 (c) 25 (d) 30
10. If  $n$  is an odd natural number,  $3^{2n} + 2^{2n}$  is always divisible by  
 (a) 13 (b) 5  
 (c) 17 (d) 19
11. For what values of  $x$ ,  $2^x \times 5^x$  ends in 5?  
 (a) 0 (b) 1  
 (c) 2 (d) No value of  $x$
12. Which of the following is a terminating decimal?  
 (a)  $\frac{4}{7}$  (b)  $\frac{3}{7}$   
 (c)  $\frac{2}{3}$  (d)  $\frac{1}{2}$
13. LCM of two co-primes (say  $x$  and  $y$ ) is \_\_\_\_\_.  
 (a)  $x + y$  (b)  $x - y$   
 (c)  $xy$  (d)  $\frac{x}{y}$
14. HCF of two co-primes (say  $x$  and  $y$ ) is \_\_\_\_\_.  
 (a)  $x$  (b)  $y$   
 (c)  $xy$  (d) 1
15. If we apply Euclid's division lemma for two numbers 15 and 4, then we get,  
 (a)  $15 = 4 \times 3 + 3$  (b)  $15 = 4 \times 2 + 7$   
 (c)  $15 = 4 \times 1 + 11$  (d)  $15 = 4 \times 4 + (-1)$

## Level 2

16. Given that the units digits of  $A^3$  and  $A$  are the same, where  $A$  is a single digit natural number. How many possibilities can  $A$  assume?  
 (a) 6 (b) 5  
 (c) 4 (d) 3
17. If the product of two irrational numbers is rational, then which of the following can be concluded?  
 (a) The ratio of the greater and the smaller numbers is an integer.  
 (b) The sum of the numbers must be rational.  
 (c) The excess of the greater irrational number over the smaller irrational number must be rational.  
 (d) None of these



18. The LCM and HCF of two numbers are equal, then the numbers must be \_\_\_\_\_.  
 (a) prime (b) co-prime  
 (c) composite (d) equal
19. Which of the following is/are always true?  
 (a) Every irrational number is a surd.  
 (b) Any surd of the form  $\sqrt[n]{a} + \sqrt[n]{b}$  can be rationalised by a surd of the form  $\sqrt[n]{a} - \sqrt[n]{b}$ , where  $\sqrt[n]{a}$  and  $\sqrt[n]{b}$  are surds.  
 (c) Both (a) and (b)  
 (d) Neither (a) nor (b)
20. The sum of LCM and HCF of two numbers is 1260. If their LCM is 900 more than their HCF, find the product of two numbers.  
 (a) 203400 (b) 194400  
 (c) 198400 (d) 205400
21. The following sentences are the steps involved in finding the HCF of 29 and 24 by using Euclid's

division algorithm. Arrange them in sequential order from first to last.

- (A)  $5 = 1 \times 5 + 0$   
 (B)  $29 = 24 \times 1 + 5$   
 (C)  $24 = 5 \times 4 + 1$   
 (a) BAC (b) ABC  
 (c) BCA (d) CAB

22. The following are the steps involved in finding the LCM of 72 and 48 by prime factorization method. Arrange them in sequential order from first to last.  
 (A)  $72 = 2^3 \times 3^2$  and  $48 = 2^4 \times 3^1$   
 (B)  $\text{LCM} = 24 \times 32$   
 (C) All the distinct factors with highest exponents are 24 and 32  
 (a) ABC (b) ACB  
 (c) CAB (d) BCA

### Level 3

23. Find the remainder when the square of any prime number greater than 3 is divided by 6.  
 (a) 1 (b) 3  
 (c) 2 (d) 4
24. If  $\text{HCF}(72, q) = 12$  then how many values can  $q$  take? (Assume  $q$  be a product of a power of 2 and a power of 3 only)  
 (a) 1 (b) 2  
 (c) 3 (d) 4
25. Find the HCF of 120 and 156 using Euclid's division algorithm.  
 (a) 18 (b) 12  
 (c) 6 (d) 24
26. The HCF of the polynomials  $(x^2 - 4x + 4)(x + 3)$  and  $(x^2 + 2x - 3)(x - 2)$  is  
 (a)  $x + 3$   
 (b)  $x - 2$   
 (c)  $(x + 3)(x - 2)$   
 (d)  $(x + 3)(x - 2)^2$

27.  $\sqrt{3 + \sqrt{5}} =$

- (a)  $\sqrt{2} + 1$  (b)  $\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$   
 (c)  $\sqrt{\frac{7}{2}} - \sqrt{\frac{1}{2}}$  (d)  $\sqrt{\frac{9}{2}} - \sqrt{\frac{3}{2}}$

28. The value of  $(37)^{3^x} - (33)^{3^x}$  ends in \_\_\_\_\_. ( $x \in \mathbb{N}$ ).  
 (a) 4 (b) 6  
 (c) 0 (d) Either (a) or (b)
29.  $P = 2(4)(6)\dots(20)$  and  $Q = 1(3)(5)\dots(19)$ . What is the HCF of  $P$  and  $Q$ ?  
 (a)  $3^3 \cdot 5 \cdot 7$  (b)  $3^4 \cdot 5$   
 (c)  $3^4 \cdot 5^2 \cdot 7$  (d)  $3^3 \cdot 5^2$
30. The LCM and the HCF of two numbers are 1001 and 7 respectively. How many such pairs are possible?  
 (a) 0 (b) 1  
 (c) 2 (d) None of these



31. There are 96 apples and 112 oranges. These fruits are packed in boxes in such a way that each box contains fruits of the same variety, and every box contains an equal number of fruits. Find the minimum number of boxes in which all the fruits can be packed.
- (a) 12 (b) 13  
(c) 14 (d) 15
32. Two runners A and B are running on a circular track. A takes 40 seconds to complete every round and B takes 30 seconds to complete every round. If they start simultaneously at 9:00 am, then which of the following is the time at which they can meet at the starting point?
- (a) 9:05 am (b) 9:10 am  
(c) 9:15 am (d) 9:13 am
33. In how many ways can 1500 be resolved into two factors?
- (a) 18 (b) 12  
(c) 24 (d) 36
34. Four bells toll at intervals of 10 seconds, 15 seconds, 20 seconds and 30 seconds respectively. If they toll together at 10:00 am at what time will they toll together for the first time after 10 am?
- (a) 10:01 am (b) 10:02 am  
(d) 10:00:30 am (d) 10:00:45 am
35. If  $a = \sqrt[8]{6} - \sqrt[8]{5}$ ,  $b = \sqrt[8]{6} + \sqrt[8]{5}$ ,  $c = \sqrt[6]{6} + \sqrt[6]{5}$ ,  $d = \sqrt[4]{6} + \sqrt[4]{5}$ , and  $e = \sqrt{6} + \sqrt{5}$ , then which of the following is a rational number?
- (a)  $abcde$  (b)  $abde$   
(c)  $ab$  (d)  $cd$
36. Find the units digit of  $(12)^{3^x} + (18)^{3^x}$  ( $x \in \mathbb{N}$ ).
- (a) 2 (b) 8  
(c) 0 (d) Cannot be determined
37. If  $X = 28 + (1 \times 2 \times 3 \times 4 \times \dots \times 16 \times 28)$  and  $Y = 17 + (1 \times 2 \times 3 \times \dots \times 17)$ , then which of the following is/are true?
- (A)  $X$  is a composite number  
(B)  $Y$  is a prime number  
(C)  $X - Y$  is a prime number  
(D)  $X + Y$  is a composite number
- (a) Both (A) and (D)  
(b) Both (B) and (C)  
(c) Both (B) and (D)  
(d) Both (A) and (B)
38.  $P$  is the LCM of 2, 4, 6, 8, 10,  $Q$  is the LCM of 1, 3, 5, 7, 9 and  $L$  is the LCM of  $P$  and  $Q$ . Then, which of the following is true?
- (a)  $L = 21P$  (b)  $L = 4Q$   
(c)  $L = 63P$  (d)  $L = 16Q$
39. The LCM and the HCF of two numbers are 144 and 12 respectively. How many such pairs of numbers are possible?
- (a) 0 (b) 1  
(c) 2 (d) None of these
40. Two bells toll in every 45 seconds and 60 seconds. If they toll together at 8:00 am, then which of the following is the probable time at which they can toll together?
- (a) 8:55 am (b) 8:50 am  
(c) 8:45 am (d) 8:40 am



## TEST YOUR CONCEPTS

### Very Short Answer Type Questions

- |                      |                     |
|----------------------|---------------------|
| 1. False             | 6. 16               |
| 2. $0.\overline{06}$ | 7. False            |
| 3. 6                 | 8. prime number     |
| 4. irrational        | 9. 15625            |
| 5. True              | 10. No value of $x$ |

### Short Answer Type Questions

- |          |                                                                    |
|----------|--------------------------------------------------------------------|
| 11. 24   | 15. (i), (ii) are non-terminating and (iii), (iv) are terminating. |
| 13. 1080 |                                                                    |

### Essay Type Questions

- |                                          |                         |
|------------------------------------------|-------------------------|
| 16. 279                                  | 18. 9:15 am             |
| 17. $2^4 \times 3^3 \times 5^2 \times 7$ | 20. $\frac{23091}{990}$ |

## CONCEPT APPLICATION

### Level 1

- |         |         |         |         |         |        |        |        |        |         |
|---------|---------|---------|---------|---------|--------|--------|--------|--------|---------|
| 1. (c)  | 2. (b)  | 3. (c)  | 4. (b)  | 5. (a)  | 6. (a) | 7. (d) | 8. (b) | 9. (c) | 10. (a) |
| 11. (d) | 12. (d) | 13. (c) | 14. (d) | 15. (a) |        |        |        |        |         |

### Level 2

- |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 16. (b) | 17. (d) | 18. (d) | 19. (d) | 20. (b) | 21. (c) | 22. (b) |
|---------|---------|---------|---------|---------|---------|---------|

### Level 3

- |         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 23. (a) | 24. (b) | 25. (b) | 26. (c) | 27. (c) | 28. (d) | 29. (c) | 30. (c) | 31. (b) | 32. (b) |
| 33. (b) | 34. (a) | 35. (b) | 36. (c) | 37. (a) | 38. (a) | 39. (c) | 40. (d) | 41. (c) | 42. (a) |
| 43. (b) | 44. (a) | 45. (c) |         |         |         |         |         |         |         |



## CONCEPT APPLICATION

### Level 1

1. (i) Given expression is in the form of  $a^2 - b^2$ .  
(ii)  $a^{2n} - b^{2n}$  is divisible by both  $(a - b)$  and  $(a + b)$ .
2. Apply trial and error method.
3. Apply trial and error method.
4. (i) Express 432 and 504 as the product of prime factors and proceed.  
(ii) Write 432 and 504 as the product of prime factors.  
(iii) HCF is the product of common prime factors with least exponents.
5. For any natural number  $n$ ,  $6^n$  and  $5^n$  end with 6 and 5 respectively.
6. (i) Apply the method of prime factorization for finding the HCF of the given numbers.  
(ii) Then find the possibilities for  $q$ .
7. (i) Recall the concepts of rational and irrational numbers.  
(ii) Recall the concept of R.F.
8. (i) Multiply and divide  $3 + \sqrt{5}$  by 2.  
(ii) Convert the expression  $\sqrt{x + 2\sqrt{y}}$  in the form of  $\sqrt{(\sqrt{m})^2 + (n)^2 + 2(mn)}$ .
9. (i) First of all find the HCF of 720 and 405.  
(ii) The required number is  $\frac{720}{\text{HCF}} + \frac{405}{\text{HCF}}$ .
10. (i)  $3^{2n} + 2^{2n} = 9^n + 4^n$ .  
(ii)  $a^n + b^n$  is divisible by  $(a + b)$  when  $n$  is odd.
11. For no value of  $x$ ,  $2^x \times 5^x$  ends in 5.
12. Only  $\frac{1}{2}$  is terminating decimal.
13. LCM of two co-primes is their product.
14. HCF of two co-primes is 1.
15.  $15 = 4 \times 3 + 3$ .

### Level 2

16. (i) Write cubes of 1 to 9.  
(ii) Then check their unit digits.
17. Recall the concept of rational numbers and irrational numbers.
18. (i) Suppose the numbers to be  $ka$  and  $kb$  where  $k$  is their HCF and  $a$  and  $b$  are co-primes.  
(ii) Check from the options.
19. (i) Recall the concepts of surds and irrational numbers.  
(ii) Recall the concepts of R.F.
20. Form the equations in LCM and HCF and solve for LCM and HCF.
21. BCA is the required sequential order.
22. ACB is the required sequential order.

### Level 3

23. Any prime number greater than 3 is in the form  $6k \pm 1$ , where  $k$  is a natural number.
24. HCF is the factor of LCM.
25. (i) Write  $156 = 1 \times 120 + 36$  and proceed further.  
(ii)  $156 = 120 \times 1 + 36$   
 $120 = 36 \times 3 + 12$ .  
(iii) Use division algorithm until we get zero as remainder.



26. Factorize the given expressions.

27. Apply trial and error method.

28. For all  $x \in N$ ,

$(37)^{3^x}$  ends in 3 or 7 and  $(33)^{3^x}$  ends in 7 or 3.

If  $(37)^{3^x}$  ends in 3, then  $(33)^{3^x}$  ends 7.

In this case  $(37)^{3^x} - (33)^{3^x}$  ends in 6 (1)

If  $(37)^{3^x}$  ends in 7, then  $(33)^{3^x}$  ends in 3.

In this case,  $(37)^{3^x} - (33)^{3^x}$  ends in 4 (2)

From Eqs. (1) and (2), option (d) follows.

29. In  $P$ , the prime numbers that occur are 2, 3, 5, 7.

In  $Q$ , there are no 2s. So the HCF of  $P$ ,  $Q$  has only 3s, 5s, 7s.

The 3's in  $P$  come from 6, 12, 18, i.e.,  $3^4$  is the greatest power of 3 that is a factor of  $P$ , while for  $Q$  the 3s come from 3, 9, 15, i.e.,  $3^4$  is also a factor of  $Q$ .

Similarly,  $5^2$  is the greatest power of 5 that for both  $P$  and  $Q$  and  $7^1$  is the greatest power of 7 for both  $P$  and  $Q$ .

$\therefore$  The HCF of  $P$ ,  $Q$  is  $3^4(5^2)(7)$ .

30. Given  $\text{LCM} = 1001$  and  $\text{HCF} = 7$ .

Let the two numbers be  $x$  and  $y$ .

$\therefore x = 7a$  and  $y = 7b$ , where  $a$  and  $b$  are co-primes.

We have,  $x \times y = \text{LCM} \times \text{HCF}$

$$7a \times 7b = 1001 \times 7$$

$$ab = 143 \Rightarrow (a, b) = (1, 143) \text{ or } (11, 13)$$

$\Rightarrow$  There are two pairs of numbers.

31. For number of boxes to be the least, the number of fruits in each box should be maximum.

As per the given data, the number of fruits in each box is equal to the HCF of 96 and 112, i.e., 16

$$\therefore \text{Minimum number of boxes} = \frac{96}{16} + \frac{112}{16} = 13.$$

32. LCM of 40 seconds and 30 seconds is 120 seconds, i.e., 2 minutes.

$\therefore$  A and B meet in every 2 minutes.

$\therefore$  Option (b) follows.

33. Number of ways in which 1500 can be resolved into two factors worked out using the concept given by

Number of factors of 1500

$$2$$

$$1500 = 3 \times 5^3 \times 2^2.$$

$$\text{Number of factors of } 1500 = (1+1)(3+1)(2+1) = 24.$$

Number of ways in which 1500 can be resolved into two factors  $= \frac{24}{2} = 12$ .

34. The time interval between simultaneous tolling of the bells = LCM (10, 15, 20, 30) seconds = 60 seconds = 1 minute.

Hence the bells will toll together again for the first time after 10:00 am at 10:01 am.

35. Given that,  $a = \sqrt[8]{6} - \sqrt[8]{5}$ ,  $b = \sqrt[8]{6} + \sqrt[8]{5}$

$$c = \sqrt[6]{6} + \sqrt[6]{5}$$

$$d = \sqrt[4]{6} + \sqrt[4]{5}, e = \sqrt{6} + \sqrt{5}$$

By inspection, we can reject  $ab$ ,  $cd$ .

$$ab = \sqrt[4]{6} + \sqrt[4]{5}$$

$$\therefore abd = \sqrt{6} - \sqrt{5} \text{ and } abde = 6 - 5 = 1.$$

$\therefore abde$  is a rational number.

36. For all  $x \in N$ ,

$(12)^{3^x}$  ends in either 8 or 2 and  $(18)^{3^x}$  ends in either 2 or 8

If  $(12)^{3^x}$  ends in 8, then  $(18)^{3^x}$  ends in 2.

If  $(12)^{3^x}$  ends in 2, then  $(18)^{3^x}$  ends in 8.

$\therefore (12)^{3^x} + (18)^{3^x}$  ends in 0 only.

37.  $X = 28 + (1 \times 2 \times 3 \times \dots \times 16 \times 28)$

$$\Rightarrow X = 28 [1 + (1 \times 2 \times 3 \times \dots \times 16)]$$

$\therefore X$  is a composite number.

$$Y = 17 + (1 \times 2 \times 3 \times 4 \times \dots \times 17) = 17 [1 + (1 \times 2 \times \dots \times 16)]$$

$\therefore Y$  is a composite number.

$$\text{Now, } X - Y = [1 + (1 \times 2 \times 3 \times \dots \times 16)] (28 - 17) = [1 + (1 \times 2 \times 3 \times \dots \times 16)] (11)$$

$\therefore X - Y$  is a composite number.

$$X + Y = [1 + (1 \times 2 \times 3 \times \dots \times 16)] (28 + 17)$$

$\therefore X + Y$  is a composite number.

$\therefore$  Option (a) follows.

38.  $P$  is the LCM of 2, 4, 6, 8, 10.  $\therefore P = 3(8)(5)$

$Q$  is the LCM of 1, 3, 5, 7, 9.

$$\therefore Q = 5(7)(9)$$

$L$  is the LCM of  $P, Q$

$$\therefore L = 3(8)(5)21 \text{ or } 5(7)(9)8, \text{ i.e., } 21P \text{ or } 8Q.$$

39. Given  $\text{LCM} = 144$  and  $\text{HCF} = 12$

Let the two numbers be  $x$  and  $y$ .

$$\therefore x = 12a \text{ and } y = 12b$$

Where  $a$  and  $b$  are co-primes.

We have,  $x \times y = \text{LCM} \times \text{HCF}$

$$12a \times 12b = 144 \times 12$$

$$ab = 12 \Rightarrow (a, b) = (1, 12) \text{ or } (3, 4)$$

There are two pairs of numbers.

40. LCM of 45 seconds and 60 seconds is 180 seconds, i.e., 3 minutes.

$\therefore$  They toll together in every 3 minutes

$\therefore$  Option (c) follows.

