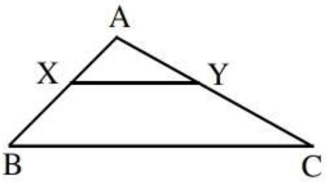
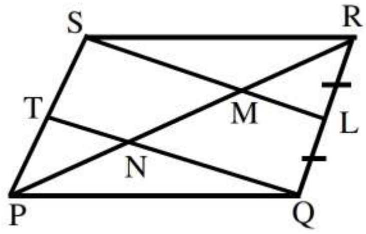
6. TRIANGLES

1. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}, AY = 5cm$ and YC = 9cm, then that whether XY and BC parallel or not



[Sol. : No, XY is not parallel to BC]

2. In the given figure, PQRS is a parallelogram and L is the mid - point of RQ prove that M is the point of trisection of PR and SL.



[Sol. : Through Q, draw a line QNT parallel to LS intersecting PR and PS at N and T respectively.

From ΔRQN, we have :

ML || NQ

So, $\frac{RL}{LQ} = \frac{RM}{MN}$ (By BPT)

But RL = LQ (L is mid - point of RQ, given)

So, RM = MN ....(i)

Now, QTSL is a parallelogram, because SL || QT and QL || TS.

(Opposite sides of parallelogram)

So, QL = TS

Hence, TS = PT (Because L is the mid - point of QR)

Now from ΔPSM, we have :

TN || SM.

So, $\frac{PT}{TS} = \frac{PN}{MN}$ (By BPT)

But PT = TS (Already shown)

So, PN = MN ....(ii)

Hence, from (i) and (ii),

PN = MN = RM

Thus, $RM = \frac{1}{3}PR$ ....(iii)

Also, $\Delta PNT \cong \Delta RML$ (By SAS)

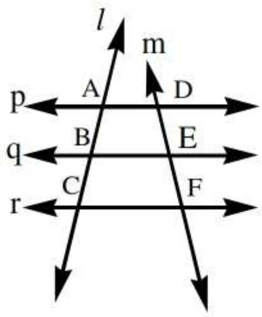
So, TN = ML (CPCT)

$\Rightarrow$ $ML = \frac{1}{2}SM$ (Since $TN = \frac{1}{2}SM$)

$\Rightarrow$ $ML = \frac{1}{3}SL$ ....(iv)

From (iii) and (iv), we get that M is the point of trisection of PR and SL.]

3. Three parallel lines p, q and r are intersected by two transversals l and m at A, B, C and D, E, F respectively as a shown in the figure. Prove that $\frac{\text{AB}}{\text{BC}} = \frac{\text{DE}}{\text{EF}}$.



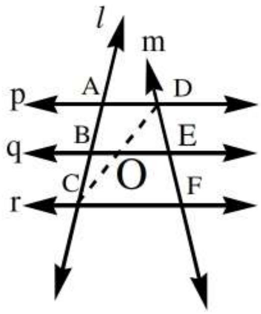
[**Ans :** **Given :** In the given figure.

p || q || r

To prove : $\frac{AB}{BC} = \frac{DE}{EF}$

Construction : Join CD which intersects BE at O.

Proof : In ΔACD, BO || AD.



So, $\frac{AB}{BC} = \frac{DO}{OC}$ ....(i) [BY BPT]

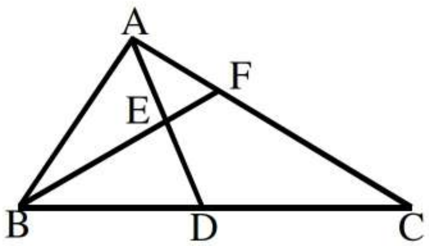
Now, in ΔDCF, OE || CF.

So, $\frac{DE}{EF} = \frac{DO}{OC}$ ....(ii) [By BPT]

From (i) and (ii), we get

$\frac{AB}{BC} = \frac{DE}{EF}$, Proved.]

4. In the figure, AD is median of ΔABC and E is the mid - point of AD. If BE produced to meet AC at F, then prove that $AF = \frac{1}{3} AC$.



[Ans : In the figure,

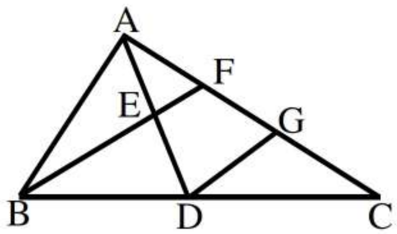
Draw DG ∥ BF.

In ΔADG, we have :

AE = ED (Given)

EF ∥ DG (DG ∥ BF)

So, by BPT, $\frac{AE}{ED} = \frac{AF}{FG}$



$\Rightarrow \frac{AE}{AE} = \frac{AF}{FG}$ (AE = ED)

$\Rightarrow$ AF = FG ....(1)

Similarly, in ΔCBF, we have :

BD = DC

and DG ∥ BF.

So, by BPT,

$$\frac{CG}{FG} = \frac{CD}{BD}$$

$\Rightarrow \frac{CG}{FG} = \frac{CD}{CD}$ (CD = BD)

$\Rightarrow$ CG = FG ....(2)

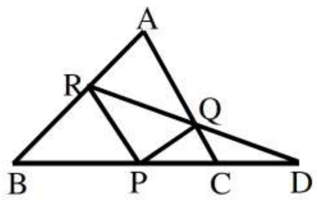
From (1) and (2),

AF = FG = CG

Also, AC = AF + FG + GC

So, $AF = \frac{1}{3} AC$.]

5. In the given figure, PQ || BA and PR || CA. If PD = 12cm, find BD × CD.



[Ans : In ΔDBR, we get

$\frac{PD}{BD}=\frac{DQ}{DR}$ (Because PQ || BA, corrollary of BPT) ......(1)

In ΔDPR, we get $\frac{CD}{PD}=\frac{DQ}{DR}$ (Because QC || RP, corrollary of BPT) ....(2)

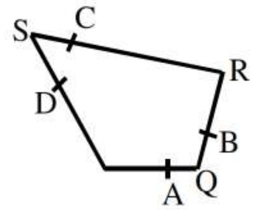
From (1) and (2), we get :

$\frac{PD}{BD}=\frac{CD}{PD}$

$\Rightarrow$ $PD^2 = BD \times CD$

$\Rightarrow$ $BD \times CD = 12 \times 12 = 144\ cm^2$.]

6. In the figure, A, B, C and D are points on the sides of a quadrilateral PQRS such that these points divide the sides PQ, RQ, RS and PS in the ratio 2:1. Prove that ABCD is a parallelogram.



[Ans : Join PR.

In ΔQPR, $\frac{QA}{PA}=\frac{QB}{BR}=1:2$

So, by converse of BPT,

AB || PR ....(1)

Similarly, in ΔSRP, we get

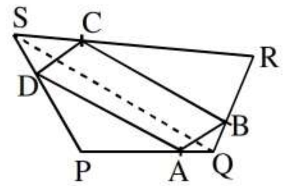
CD || PR ....(2)

From (1) and (2), we get

AB || CD ....(3)

Similarly, by joining QS, we can get

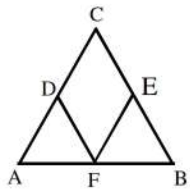
BC || AD ....(4)



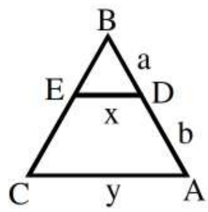
From (3) and (4), opposite sides of quadrilateral ABCD are parallel.

So, ABCD is a parallelogram.]

7. In the figure, DF ∥ BC and $\frac{AD}{DC} = \frac{CE}{BE}$. Prove that FDCE is a parallelogram.



8. In the given figures, DE ∥ AC. Find *x*.



[Ans : DE ∥ AC (Given)

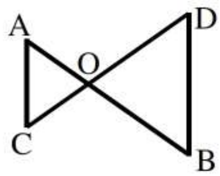
So, $\angle E = \angle C$ ans $\angle D = \angle A$ (corresponding angles)

Therefore, $\Delta BDE \sim \Delta BAC$ (AAA similarly criterion)

So, $\frac{BD}{AB} = \frac{DE}{AC} \Rightarrow \frac{a}{a+b} = \frac{x}{y}$

$\Rightarrow \quad x = \frac{ay}{a+b}$.]

9. $\Delta OCA \sim \Delta OBD$, then prove that AC ∥ BD.



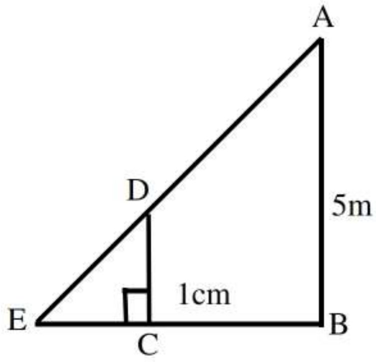
[Ans : $\Delta OCA \sim \Delta ODB$ (Given)

So, $\angle C = \angle D$ (corresponding angles)

But, they are alternate interior angles.

So, AC ∥ BD (Proved)]

10. A girl of height 100cm is walking away from the base of a lamp post at a speed of 1.9 m/s. If the lamp is 5cm above the ground, find the length of her shadow after 4 seconds.



[Ans : Speed of girl = 1.9 m/second

After 4 seconds, distance covered by the girl

$= 1.9 \times 4$ m = 7.6 m

AB = Lamp post = 5 m

CD = Height of girl

= 100 cm = 1 m

= 1 m.

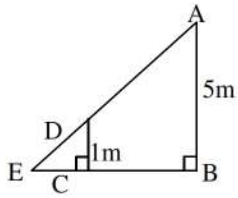
EC = Shadow of girl = ?

$\angle B = \angle C = 90°$ [Each]

So, $\angle E = \angle E$ [Common angle]

Now, $\Delta ABE \sim \Delta DCE$ [AA similarly]

Hence, $\frac{AB}{CD} = \frac{BE}{EC}$



$\Rightarrow \frac{5}{1} = \frac{7.6 + EC}{EC}$

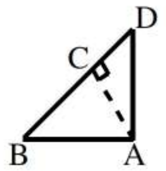
or 5EC = 7.6 + EC

$\Rightarrow$ 4EC = 7.6

$\Rightarrow EC = \frac{7.6}{4} = 1.9m$

Hence, length of shadow is 1.9 m.]

11. If in the figure given below, $\Delta ABC \sim \Delta CAD$, then prove that $AD^2 = BD \times CD$.

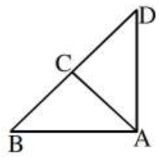


[Sol. : $\Delta ABD \sim \Delta CAD$ (Given)

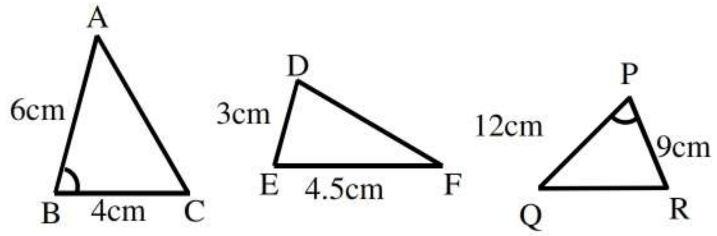
So, $\frac{AD}{BD} = \frac{CD}{AD}$

$\Rightarrow$ $AD \times AD = BD \times CD$

$\Rightarrow$ $AD^2 = BD \times CD$, proved.]



12. State which of the two triangles given in the figure are similar. Also, state the similarity criterion used.



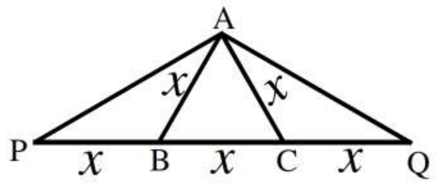
[Sol. : Since $\frac{6}{4} = \frac{4.5}{3} = \frac{3}{2}$ (By SAS similarity criterion)

So, $\Delta ABC \sim \Delta FED$.

13. In the given figure, ΔABC is an equilateral triangle, whose each side measures x units. P and Q are two points on BC produced such that PB = BC = CQ. Prove that

a) $\frac{PQ}{PA} = \frac{PA}{PB}$

b) $PA^2 = 3x^2$



[Sol. : a) In ΔPAB, PB = AB

So, $\angle APB = \angle PAB$ (Angles opposite the equal sides)

Also, $\angle ABP = 180° - 60° = 120°$.

So, $\angle APB = \angle PAB = \frac{1}{2}(180° - 120°) = 30°$

Similarly, $\angle QAC = \angle QCA = 30°$

So, $\angle PAQ = \angle PAB + \angle BAC + \angle QAC$

$= 30° + 60° + 30° = 120°$.

Now, in ΔPQA and ΔPAB, we have :

$\angle APQ = \angle APB$ (Each 30°)

$\angle PAQ = \angle PBA$ (Each 120°)

and $\angle PQA = \angle PAB$ (Each 30°)]

So, $\Delta PQA \sim \Delta PAB$ (By AAA similarity criterion)

Hence, $\frac{PQ}{PA} = \frac{PA}{PB}$ (Proved)

b) PQ = 3x

So, from $\frac{PQ}{PA} = \frac{PA}{PB}$, we have

$PA^2 = PQ \times PB$

$\Rightarrow$ $PA^2 = 3x \times x = 3x^2$. (Proved)

14. Prove that if two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then the two triangles are similar.

(OR)

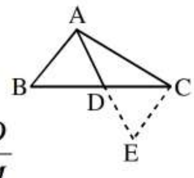
In ΔABC, AD is the median to BC and in ΔPQR, RM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, prove that $\Delta ABC \sim \Delta PQR$.

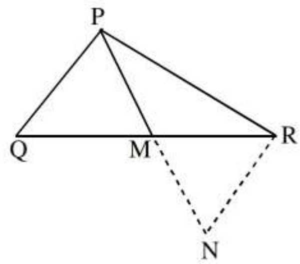
[Proof: Produce AD to E such that AD = DE and PM to N such that PM = MN. Join CE and RN.

Now, $\Delta ABD \cong \Delta ECD$ (SAS)

and $\Delta PQM \cong \Delta NRM$ (SAS)

So, AB = CE and PQ = RN (By CPCT)





Now, in ΔACE and ΔPRN, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow$ $\frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ ($\because AB = CE$ and PQ = RN)

So, $\Delta ACE \sim \Delta PRN$ (By SSS similarity criterion)

So, $\angle CAD = \angle RPM$ ....(1)

Again, in ΔBAD and ΔQPM,

$\angle BAD = \angle CED$ ($\because \Delta ABD \cong \Delta ECD$)

$\angle QPM = \angle RNM$ ($\because \Delta PQM \cong \Delta NRM$)

$\angle AEC = \angle PNR$ ($\because \Delta AEC \sim \Delta PNR$)

Therefore, $\angle BAD = \angle QPM$ ....(2)

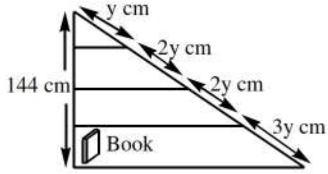
Adding (1) and (2), $\angle A = \angle P$

Now, in ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$

So, $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)**]**.

15. Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below.

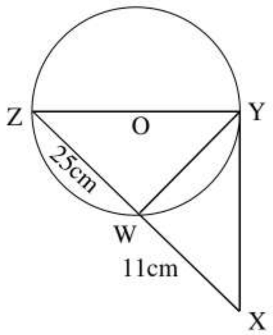


The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom - most shelf?

a) 18 cm b) 36 cm c) 54 cm d) 86.4 cm

[Ans: (c) 54 cm**]**

16. Shown below is a circle with centre O. YX is the tangent to the circle at Y.



i) Prove that $\Delta ZWY \sim \Delta ZYX$

ii) Using part i), find the length of ZY.

Show your steps and give valid reasons.

[Sol. : i) We know that $\angle ZYX = 90°$ and gives the reason that radius is always perpendicular to the tangent at the point of contact.

Write that $\angle ZWY = 90°$ and gives reason that angle in a semicircle is always 90°.

In ΔZWY and ΔZYX :

$\angle Z$ is common.

$\angle ZWY = \angle ZYX = 90°$ (using step 1 and 2)

Concludes that $\Delta ZWY \sim \Delta ZYX$ by using the AA similarity criterion.

ii) Uses above step and then relation as :

$$\frac{ZY}{ZX} = \frac{ZW}{ZY}$$

$$\Rightarrow \quad \frac{ZY}{25+11} = \frac{25}{ZY}$$

The length of ZY as 30 cm.