7. COORDINATE GEOMETRY

1. Determine the vertex which contains a right angle in ΔABC, where A(4, –2), B(7, 9), C(7, –2)

   [Ans: Vertex C]

2. If(0,0), (5,0) and (x, y) are the vertices of an equilateral triangle, find x and y.

   [Sol. : We have : $(0 - 5)^2 + (0 - 0)^2 = (5 - x)^2 + (0 - y)^2$

   $\Rightarrow \quad 25 = 25 - 10x + x^2 + y^2$

   $\Rightarrow \quad x^2 + y^2 - 10x = 0 \qquad$ .....(1)

   Also, $(x - 0)^2 + (y - 0)^2 = (5 - x)^2 + (0 - y)^2$

   $\Rightarrow \quad x^2 + y^2 = 25 - 10x + x^2 + y^2 \qquad$ .....(2)

   $\Rightarrow \quad 10x = 25 \quad \Rightarrow \quad x = \frac{25}{10} = \frac{5}{2}$

   Putting the value of $x = \frac{5}{2}$ in (1), we get

   $\frac{25}{4} + y^2 - 10 \times \frac{5}{2} = 0 \quad \Rightarrow \quad y^2 = 25 - \frac{25}{4} = \frac{75}{4}$

   $\Rightarrow \quad y = \pm \frac{5}{2}\sqrt{3}$.

   Thus $x = \frac{5}{2}$ and $y = \pm \frac{5}{2}\sqrt{3}$]

3. If a point P is equidistant from A(–7, –5) and B(–9, –1) and its abscissa is equal to its ordinate, find the coordinates of the point P.

   [Ans : P(2, 2)]

4. The centre of a circle is (3x –8, 2x - 5). If the circle passes through the point A(4,3) and the length of its diameter is $4\sqrt{13}$ units, find the value of x.

   [Ans : x = 6 (or) 2]

5. The vertices of a ΔPQR are P(–2, 0), Q(2, 0) and R(0, 2). The vertices of ΔXYZ are X(–4,0), Y(4, 0) and Z(0, 4). Check whether $\Delta PQR \sim \Delta XYZ$ or not.

[Ans : $\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{PR}{ZX} = \frac{1}{2}$

So, $\Delta PQR \sim \Delta XYZ$]

6. If the coordinates of the centre of a circle are (5,a) and two points on the circle are (9,1) and (5,5) then find the value a. Also, find the area of this circle.

[Ans : a = 1, Area of the circle = 16π sq. units]

7. With the help of distance formula, show that the points A(4,2) B(7, 5) and C(9, 7) do not form a triangle.

8. If the point P(3, 4) is equidistant from the points A(a + b, b – a) and B(a –b, a +b), then prove that 3b – 4a = 0.

9. If the line segment joining the points P and Q(3, –4) is bisected at origin find the coordinates of P.

[Hint : use mid - point formula, we get P(–3, 4)]

10. Show that the line - segments joining the points (4,2) and (–6, 4) and (–10, 5) and (8, 1) bisect each other.

[Sol. : Mid - point of the line segment joining (4, 2) and (–6, 4) is

$$= \left(\frac{4-6}{2}, \frac{2+4}{2} \right) = (-1, 3)$$

Also, mid - point of line segment joining (–10, 5) and (8, 1) is

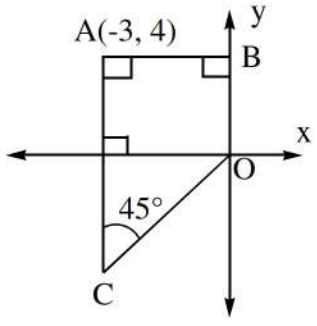
$$= \left(\frac{-10+8}{2}, \frac{5+1}{2} \right) = (-1, 3)$$

Thus, mid - point of each of the two line segments is (–1, 3), i.e., the same. So, the two line segments bisect each other.]

11. The lines segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 3x – 2y + k = 0, find the value of k

[Ans : k = – 3]

12. A quadrilateral ABOC is drawn on a coordinate grid as shown below. O is the origin and the coordinates of A are (–3, 4). $\angle OCA = 45°$.



What is the perimeter of ABOC?

a) $14+3\sqrt{2}units$ b) $17+3\sqrt{2}units$ c) $14+4\sqrt{2}units$

d) (cannot be found using the given information)

[Ans : a) $14+3\sqrt{2}$units)

13. ΔABC is a triangle such that AB : BC = 1 : 2. Point A lies on the y - axis and the coordinates of B and C are known.
Which of the following formula can DEFINITELY be used to find the coordinates of A?
i) Section formula ii) Distance formula
a) only (i) b) only (ii) c) both (i) and (ii) d) neither (i) or (ii)
[Ans : b) only (ii)**]**

14. The radius of a circle with centre at the origin is $\frac{1}{2}$ units.
Find all the points on the circle which are of the form (–y, y). Show your steps.

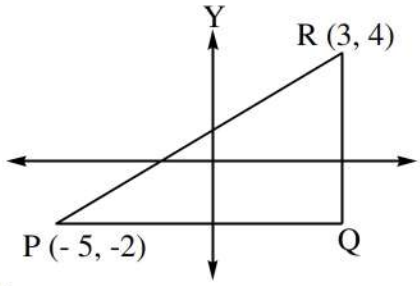
[Sol. : The distance between the origin and the point (–y, y) as $\frac{1}{2}$ units and the distance formula to write the equation as : $(-y)^2+y^2=\left(\frac{1}{2}\right)^2$

Simplifies the above equation as $2y^2=\frac{1}{4}$

Solves the above equation to get y as $\frac{1}{\sqrt{8}}$ and $\frac{-1}{\sqrt{8}}$

The points are $\left(\frac{1}{\sqrt{8}},\frac{-1}{\sqrt{8}}\right)$ and $\left(\frac{-1}{\sqrt{8}},\frac{1}{\sqrt{8}}\right)$**]**.

15. Shown below is a right triangle PQR.
Find the value of cos P. Show your work,



[Sol. : The coordiantes of Q as (3, – 2).
The distance formula and finds
PR as $\sqrt{\left(8^2+6^2\right)}=10$ units and PQ as $\sqrt{8^2=8}$ units.

Mention that $\cos p=\frac{PQ}{PR}$ and finds its value as $\frac{8}{10}=\frac{4}{5}$**]**.

16. Preeti and Arun are both driving to their respective offices from the same home. Preeti drives towards the east at an average speed of 30 km per hour for 12 minutes and then towards the south at an average speed of 60 km per hour for 3 minutes. Arun drives towards west at an average speed of 30 km per hour for 4 minutes and then towards the north at an average speed of 45 km per hour for 4 minutes.

What is the straight-line distance between Preeti's office and Arun's office? Show your steps and represent the given scenario on the coordinate plane.

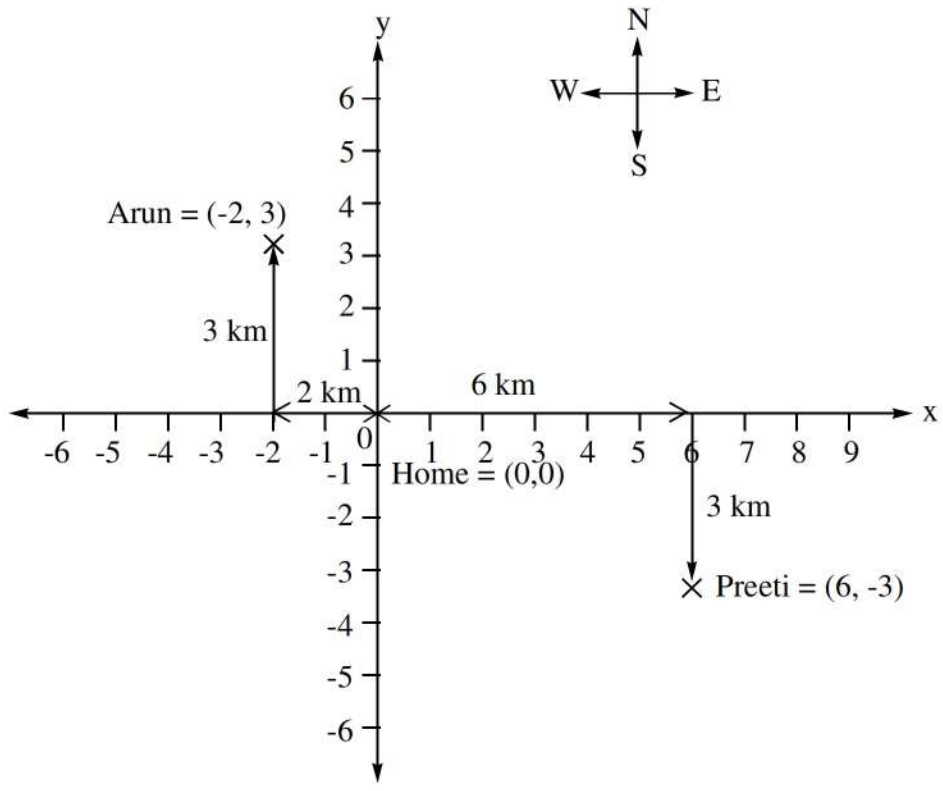
[Ans : The distance travelled by Preeti towards the east as $\frac{30\times12}{60}=6\,km$.

The distance travelled by Preeti towards the south as $\frac{60\times3}{60}=3\,km$.

The distance travelled by Aruna towards the west as $\frac{30\times4}{60}=2\,km$.

The distance travelled by Aruna towards the north as $\frac{45\times4}{60}=3\,km$

The scenario on a coordinate plane. The figure may look as follows:



Locates the coordinates of Preeti's office as (6, – 3) and the coordinates of Arun's office as (–2, 3)] .

17. Raji and Gagan are finding a treasure that is exactly on the straight line joining them.

Raji's location is at (–6, – 5) and Gagan's location is at (10, 11). The distance from the treasure to Raaji's location is three times that of the distance of Gagan's location.

Find the coordinates of the location of the treasure. Show your steps.

[Sol. : Writes that (x, y) are the coordinates of the location of the treasure that divide(– 6, – 5) and (10, 11) in the ratio m : n = 3 : 1.

Using the section formula to find (x, y) as follows:

$$\left(\frac{3(10)+1(-6)}{3+1}, \frac{3(11)+1(-5)}{3+1}\right)$$

$= (6, 7)$].

18. On a golf course, three holes A (– 6, – 1), B and C (9, – 4) lie on a straight line in that order. The distance between B and C is two times that between B and A.

Rahul strikes the ball, which is at point P (2, 3), such that it goes in the hole B.

i) Find the coordinates of hole B.

ii) Find the shortest distance covered by the ball.

Show your steps.

[Sol. : i) Writes that point B internally divides the line joining A and C in the ratio 1 : 2

Using the section formula to find the coordinates of hole B as:

$$\left(\frac{2(-6)+1(9)}{1+2}, \frac{2(-1)+1(-4)}{1+2}\right) = (-1, -2)$$

ii) The shortest distance = $\sqrt{34}$ units]

19. PQ is a line segment such that the y-coordiante of P is – 1 and Q lies on the y-axis. The mid-point of PQ is O (– 3, – 6).

Find the coordiantes of Q. Show your work.

[Sol. : The coordiates of Q is of the form (0, y) and use the mid-point formula to find the value of

y as $\left(\frac{-1+y}{2}\right) = -6$.

Solve the above equation to find the value of y as – 11 and the coordinates of Q as (0, –11)].

20. A circle with centre O (2, – 5) has a chord with end-points A (1, 2) and B. M (5, –2) is the point where the perpendicular to the chord from the centre touches AB.

Find the coordinates of point B. Show your steps with valid reasons.

[Sol. : The point M, where the perpendicular to the chord from the centre touches AB, is the mid-point of AB and writes the expressions to find the coordinates of point B as

$$\left(\frac{1+x}{2}, \frac{2+y}{2}\right) = (5, -2).$$

The coordinates of point B as (9, – 6)].