CHAPTER – 3
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

ALGEBRAIC INTERPRETATION OF PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

The pair of linear equations represented by these lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the pair of linear equations has exactly one solution.
2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.
3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solution.

S. No.	Pair of lines	Compare the ratios	Graphical representation	Algebraic interpretation
1	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution (Exactly one solution)
2	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

IMPORTANT QUESTIONS

1. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:
   (i) $5x – 4y + 8 = 0$ and $7x + 6y – 9 = 0$ (ii) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$
   (iii) $6x – 3y + 10 = 0$ and $2x – y + 9 = 0$.
2. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.
   (i) $3x + 2y = 5$; $2x – 3y = 7$ (ii) $2x – 3y = 8$; $4x – 6y = 9$
   (iii) $5x – 3y = 11$; $– 10x + 6y = –22$
3. Find the number of solutions of the following pair of linear equations:
   $x + 2y – 8 = 0$
   $2x + 4y = 16$
4. Write whether the following pair of linear equations is consistent or not.
   $x + y = 14$, $x – y = 4$
5. Given the linear equation $3x + 4y – 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is parallel lines.
6. Find the value of k so that the following system of equations has no solution:
   $3x – y – 5 = 0$, $6x – 2y + k = 0$
7. Find the value of k so that the following system of equation has infinite solutions:
   $3x – y – 5 = 0$, $6x – 2y + k = 0$
8. For which values of p, does the pair of equations given below has unique solution?
   $4x + py + 8 = 0$ and $2x + 2y + 2 = 0$

9. Determine k for which the system of equations has infinite solutions:
$4x + y = 3$ and $8x + 2y = 5k$

10. Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:
$2x – 3y + 6 = 0$; $4x – 5y + 2 = 0$

11. Find the value of k for which the system $3x + ky = 7$, $2x – 5y = 1$ will have infinitely many solutions.

12. For what value of k, the system of equations $2x – ky + 3 = 0$, $4x + 6y – 5 = 0$ is consistent?

13. For what value of k, the system of equations $kx – 3y + 6 = 0$, $4x – 6y + 15 = 0$ represents parallel lines?

14. For what value of p, the pair of linear equations $5x + 7y = 10$, $2x + 3y = p$ has a unique solution.

15. Find the value of m for which the pair of linear equations has infinitely many solutions.
$2x + 3y – 7 = 0$ and $(m – 1)x + (m + 1)y = (3m – 1)$

16. For what value of p will the following pair of linear equations have infinitely many solutions?
$(p – 3)x + 3y = p$; $px + py = 12$

17. For what value of k will the system of linear equations has infinite number of solutions?
$kx + 4y = k – 4$, $16x + ky = k$

18. Find the values of a and b for which the following system of linear equations has infinite number of solutions:
$2x – 3y = 7$, $(a + b) x – (a + b – 3) y = 4a + b$

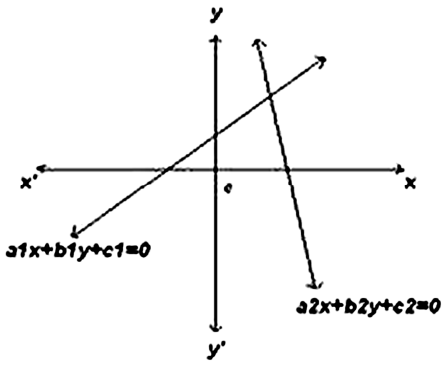
19. For what value of k will the equations $x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represent coincident lines?

20. For what value of k, the following system of equations $2x + ky = 1$, $3x – 5y = 7$ has (i) a unique solution (ii) no solution

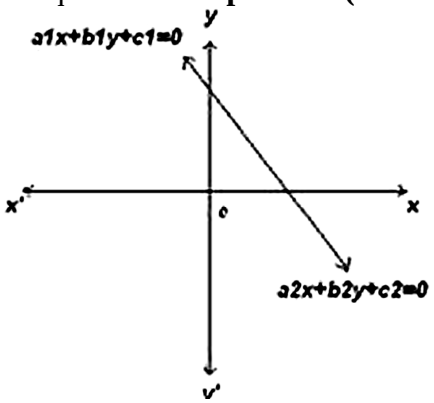
GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

The graph of a pair of linear equations in two variables is represented by two lines.

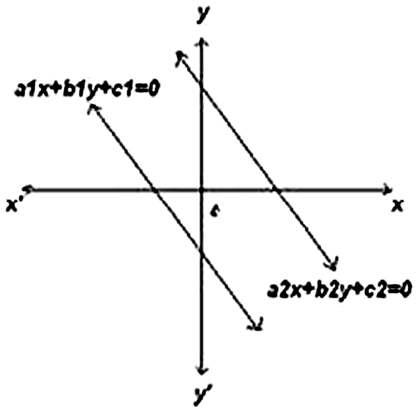
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



2. If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.



IMPORTANT QUESTIONS

Solve the equation graphically: x + 3y = 6 and 2x – 3y = 12.

Solution: Given that

$$x+3y=6 \Rightarrow 3y=6-x \Rightarrow y=\frac{6-x}{3}$$

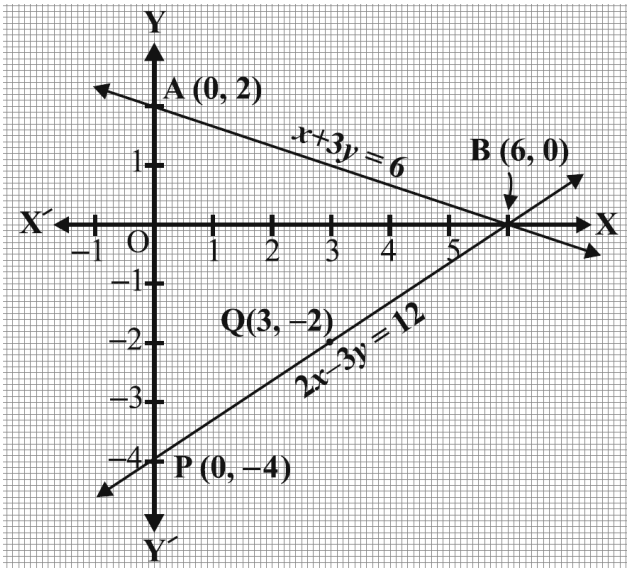
x	0	3	6
y	2	1	0

and $2x-3y=12 \Rightarrow 3y=2x-12 \Rightarrow y=\frac{2x-12}{3}$

x	0	3	6
y	-4	-2	0

Now plot the points and join the points to form the lines AB and PQ as shown in graph

Since point B(6, 0) common to both the lines AB and PQ. Therefore, the solution of the pair of linear equations is x = 6 and y = 0



Questions for Practice

1. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 3x + 4y = 12; y = 2

2. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 2x – 5 = 0, y + 4 = 0.

3. Draw the graphs of the equations 4x – y – 8 = 0 and 2x – 3y + 6 = 0.
Also, determine the vertices of the triangle formed by the lines and x-axis.

4. Solve the following system of linear equations graphically:3x – 2y – 1 = 0; 2x – 3y + 6 = 0.
Shade the region bounded by the lines and x-axis.

5. Solve graphically: x + 4y = 10, y – 2 = 0

6. Solve graphically: 2x – 3y = 6, x – 6 = 0

7. Solve the following system of equations graphically: 3x – 5y + 1 = 0, 2x – y + 3 = 0.
Also find the points where the lines represented by the given equations intersect the x-axis.

8. Solve the following system of equations graphically: x – 5y = 6, 2x – 10y = 10
Also find the points where the lines represented by the given equations intersect the x-axis.

9. Solve the following pair of linear equations graphically: x + 3y = 6; 2x – 3y = 12
Also find the area of the triangle formed by the lines representing the given equations with y-axis.

MCQ QUESTIONS (1 mark)

1. The pair of equations y = 0 and y = –7 has
(a) one solution (b) two solution (c) infinitely many solutions (d) no solution

2. The pair of equations x = a and y = b graphically represents the lines which are
(a) parallel (b) intersecting at (a, b)
(c) coincident (d) intersecting at (b, a)

3. The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have no solution is
(a) 3 (b) – 3 (c) – 12 (d) no value

4. The pair of equations 5x – 15y = 8 and 3x – 9y = 24/5 has
(a) infinite number of solutions (b) unique solution
(c) no solution (d) one solution

5. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have
(a) infinite number of solutions (b) unique solution
(c) no solution (d) one solution

6. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the numbers get reversed. The number is
(a) 36 (b) 72 (c) 63 (d) 25

7. The pair of equations 3x + 4y = 18 and $4x + \frac{16}{3}y = 24$ has
(a) infinite number of solutions (b) unique solution
(c) no solution (d) cannot say anything

8. If the pair of equations 2x + 3y = 7 and $kx + \frac{9}{2}y = 12$ have no solution, then the value of k is:
(a) $\frac{2}{3}$ (b) – 3 (c) 3 (d) $\frac{3}{2}$

9. If 2x + 3y = 0 and 4x – 3y = 0, then x + y equals:
(a) 0 (b) – 1 (c) 1 (d) 2

10. If 31x + 43y = 117 and 43 + 31y = 105, then value of x – y is:
(a) $\frac{1}{3}$ (b) – 3 (c) 3 (d) $-\frac{1}{3}$

11. If 19x – 17y = 55 and 17x – 19y = 53, then the value of x – y is:
(a) $\frac{1}{3}$ (b) – 3 (c) 3 (d) 5

12. If (6, k) is a solution of the equation 3x + y – 22 = 0, then the value of k is:
(a) 4 (b) –4 (c) 3 (d) –3

13. If 3x + 2y = 13 and 3x – 2y = 5, then the value of x + y is:
(a) 5 (b) 3 (c) 7 (d) none of these

14. If the pair of equations 2x + 3y = 5 and $5x + \frac{15}{2}y = k$ represent two coincident lines, then the value of k is:
(a) –5 (b) $\frac{-25}{2}$ (c) $\frac{25}{2}$ (d) $\frac{-5}{2}$