VERY SIMILAR
PRACTICE TEST 1

Hints and Explanations

1. (a) : As no external torque is applied to the system, the angular momentum of the system remains conserved.

$\therefore \quad L_i = L_f$

where the subscripts represent initial and final.

or $\quad I_i\omega_i = I_f\omega_f$

Substituting the given values, we get

$\therefore \quad 1 \times 100 = 2 \times 1 \times \omega_f$

$\omega_f = 50 \text{ rad s}^{-1}$ ...(i)

Initial kinetic energy, $K_i = \frac{1}{2} I_i\omega_i^2$

$= \frac{1}{2} \times 1 \times (100)^2 = 5 \times 10^3 \text{ J}$

Final kinetic energy, $K_f = \frac{1}{2} \times I_f \times \omega_f^2$

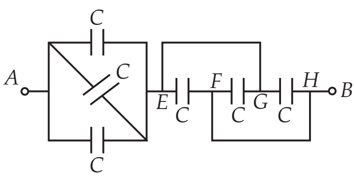
$= \frac{1}{2} \times 2 \times (50)^2$ (Using (i))

$= 2.5 \times 10^3 \text{ J}$

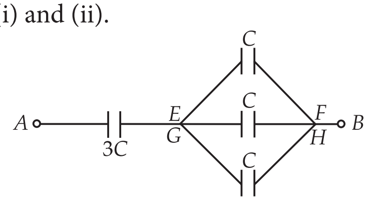
Loss in kinetic energy, $\Delta K = K_i - K_f$

$= 5 \times 10^3 - 2.5 \times 10^3 \text{ J} = 2.5 \times 10^3 \text{ J} = 2.5 \text{ kJ}$

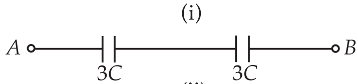
2. (a) :



The equivalent circuit diagrams are shown in the figure (i) and (ii).



(i)



(ii)

The equivalent capacitance between A and B is

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{3C}, \quad \frac{1}{C_{eq}} = \frac{2}{3C}, \quad C_{eq} = \frac{3}{2}C = 1.5\,C$$

3. (d) : As the process is cyclic, therefore, $\Delta U = 0$

According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \Delta W$ or $\Delta W = \Delta Q$

or $\quad W_1 + W_2 + W_3 + W_4 = Q_1 + Q_2 + Q_3 + Q_4$

$W_4 = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$

Substituting the given values, we get

$= (5960 - 5585 - 2980 + 3645) - (2200 - 825 - 1100)$

$= 1040 - 275 = 765 \text{ J}.$

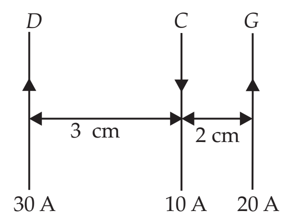
Efficiency, $\eta = \frac{\text{Net work done}}{\text{Total heat absorbed}}$

$= \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}$

$\eta = \frac{2200 - 825 - 1100 + 765}{5960 + 3645} = \frac{1040}{9605} = 0.1083$

$= 0.1083 \times 100\% = 10.83\%$

4. (c) :



The magnetic field due to wire D at wire C is

$$B_D = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{r} = \frac{10^{-7} \times 2 \times 30}{0.03} = 2 \times 10^{-4} \text{ T}$$

which is directed into the page.

The magnetic field due to wire G at C is

$$= \frac{10^{-7} \times 2 \times 20}{0.02} = 2 \times 10^{-4} \text{ T}$$

which is directed out of the page.

Therefore, the field at the position of the wire C is

$B = B_D - B_G = 2 \times 10^{-4} - 2 \times 10^{-4} = \text{zero}.$

$\therefore$ The force on 25 cm of wire C

$F = BIL \sin\theta = \text{zero}.$

5. (d) : Energy density

$$= \frac{\text{Energy}}{\text{Volume}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = [\text{ML}^{-1}\text{T}^{-2}]$$

$$\frac{\text{Force}}{\text{Area}} = \frac{[\text{MLT}^{-2}]}{[\text{L}^2]} = [\text{ML}^{-1}\text{T}^{-2}]$$

$$\frac{\text{Charge}}{\text{Volume}} \times \text{Voltage} = \frac{\text{Charge}}{\text{Volume}} \times \frac{\text{Work}}{\text{Charge}}$$

$$= \frac{\text{Work}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

$$\frac{\text{Angular momentum}}{\text{Mass}} = \frac{[ML^2T^{-1}]}{[M]} = [M^0L^2T^{-1}]$$

Therefore, dimensions of (iv) is different.

6. (b) : The flux through the stationary loop in the problem is $\phi = at(\tau - t)$

Induced emf,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}[at(\tau - t)] = -[a\tau - 2at] = (2at - a\tau)$$

The amount of heat generated in the loop during a small time interval dt is

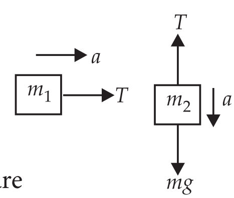
$$dQ = \frac{\varepsilon^2}{R}dt = \frac{(2at - a\tau)^2}{R}dt$$

Hence, the total heat generated is

$$Q = \int_0^\tau \frac{(2at - a\tau)^2}{R}dt = \frac{1}{R}\int_0^\tau (4a^2t^2 + a^2\tau^2 - 4a^2\tau t)dt$$

$$= \frac{1}{R}\left[\frac{4}{3}a^2t^3 + a^2\tau^2 t - \frac{4}{2}a^2\tau t^2\right]_0^\tau = \frac{1}{3}\frac{a^2\tau^3}{R}$$

7. (b) : Let a be common acceleration of the system. The free body diagrams of two blocks are as shown in the figure.



Their equations of motion are

$$T = m_1 a \quad \text{...(i)}$$

$$m_2 g - T = m_2 a \quad \text{...(ii)}$$

From (i) and (ii), we get

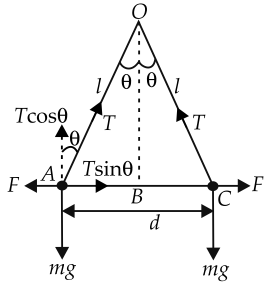
$$a = \frac{m_2 g}{m_1 + m_2} \quad \text{...(iii)}$$

Using, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \ d = 0 \times t + \frac{1}{2}\frac{m_2 g}{m_1 + m_2}t^2 \quad \text{(Using (iii))}$$

or $\quad t = \sqrt{\frac{2d(m_1 + m_2)}{m_2 g}}$

8. (a) : Figure shows equilibrium positions of the two spheres.



$\therefore \ T\cos\theta = mg$

and $T\sin\theta = F$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$$

$$\therefore \ \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$$

When charge begins to leak from both the spheres at a constant rate. Let x be the distance between them at any instant.

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg} \quad \left(\because \ \tan\theta = \frac{x}{2l}\right)$$

or $\frac{x}{2l} \propto \frac{q^2}{x^2}$ or $q^2 \propto x^3 \Rightarrow q \propto x^{3/2}$

$$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$$

or $\ v \propto x^{-1/2} \quad \left(\because \ \frac{dq}{dt} = \text{constant}\right)$

9. (b) : At the time of projection kinetic energy of the stone, $K = \frac{1}{2}mu^2$

where m is the mass of the stone and u is the velocity of the projection.

or $\ u^2 = \frac{2K}{m} = \frac{2 \times 98}{2} = 98$

Using, $v^2 = u^2 - 2gh$

$$\therefore \ h = \frac{u^2}{2g} \quad (\because \ v = 0) \quad \text{...(i)}$$

$$h = \frac{98}{2 \times 9.8} = 5 \text{ m}$$

Also, $\ K = \frac{1}{2}m\left(\sqrt{2gh}\right)^2$ (Using (i))

$$K' = \frac{1}{2}mv'^2 = \frac{1}{2}m \times \left(\sqrt{2gh'}\right)^2$$

$$\therefore \ \frac{K'}{K} = \frac{h'}{h}$$

According to the problem

$$K' = \frac{K}{2}$$

$$\frac{K}{2K} = \frac{h'}{h}$$

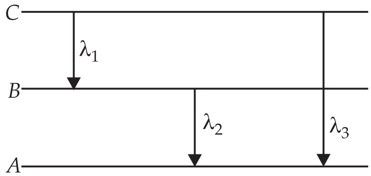
$$h' = \frac{h}{2} = \frac{5}{2} \text{ m} = 2.5 \text{ m}$$

10. (a) : R is independent of frequency.

Current becomes maximum at resonance.

$X_L = \omega L = 2\pi\upsilon L$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

11. (d) : 

As is clear from figure,

$E_3 = E_1 + E_2$

$h\upsilon_3 = h\upsilon_1 + h\upsilon_2$ or $\upsilon_3 = \upsilon_1 + \upsilon_2$

From $E_3 = E_1 + E_2$

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \text{ or } \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{\lambda_2 + \lambda_1}{\lambda_1 \lambda_2}$$

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

12. (d) : When the lift is accelerated upwards with acceleration a, then tension in the wire is

$T = m(g + a) = 1000(9.8 + 1.2) = 11000$ N

$$\text{Now, stress} = \frac{F}{A} = \frac{T}{\pi r^2} \text{ or } r^2 = \frac{T}{\pi\,(\text{stress})}$$

$$r^2 = \frac{11000 \times 7}{22 \times 1.4 \times 10^8} = \frac{1}{4 \times 10^4} \text{ or } r = \frac{1}{200}$$

$\therefore$ Minimum diameter of the wire

$$D = 2r = \frac{1}{100} = 0.01 \text{ m}$$

13. (d) : R.P. of telescope $= \dfrac{D}{1.22\,\lambda}$

It can be increased by increasing D and decreasing λ.

14. (b) : Surface energy $u = S \times 4\pi R^2$

When droplet is splitted into 1000 droplets each of radius r, then

$$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{10}$$

Here, R is the radius of bigger drop and S is the surface tension.

$\therefore$ Surface energy of all droplets

$= S \times 1000 \times 4\pi r^2 = S \times 1000 \times 4\pi(R/10)^2$

$= 10(S4\pi R^2) = 10u$

15. (b) : Heat released by 5 kg of water when its temperature fall from 20°C to 0°C is

$Q_1 = m_{\text{water}}\, s_{\text{water}}\, \Delta T$

$= 5 \times 10^3 \times 1 \times (20 - 0) = 10^5$ cal

Heat absorbed by 2 kg of ice at –20°C to increase its temperature to 0°C is

$Q_2 = m_{\text{ice}}\, s_{\text{ice}}\, \Delta T$

$= 2 \times 10^3 \times 0.5 \times 20 = 0.2 \times 10^5$ cal

So, the temperature of mixture will be 0°C.

The remaining heat

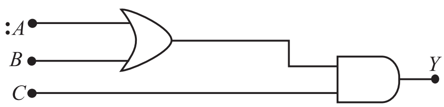
$Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal

The remaining heat will melt a mass m of ice

$$\therefore \quad m = \frac{Q}{L_f} = \frac{0.8 \times 10^5}{80} = 10^3 \text{g} = 1 \text{ kg}$$

The final mass of water in the container

$= (5 + 1)$ kg $= 6$ kg

16. (d) : 

A	B	C	$A + B$	$Y = (A + B)\cdot C$
0	1	0	1	0
0	0	1	0	0
1	0	0	1	0
1	0	1	1	1

17. (b) : Escape velocity, $v_e = \sqrt{2Rg}$

where R is the radius of the planet.

Potential energy of the body on the surface of the planet,

$U_S = -\dfrac{GMm}{R}$ (where m is the mass of the body)

Potential energy of the body at the centre of the planet,

$$U_C = -\frac{3}{2}\frac{GMm}{R}$$

If v is the velocity acquired by the body while at the centre of the planet, then

$$\frac{1}{2}mv^2 = U_S - U_C = -\frac{GMm}{R} - \left(-\frac{3GMm}{2R}\right)$$

$$\text{or} \quad \frac{1}{2}mv^2 = -\frac{GMm}{R} + \frac{3}{2}\frac{GMm}{R}$$

$$\text{or} \quad v^2 = 2\frac{GM}{R}\left(\frac{3}{2} - 1\right) = \frac{GM}{R} = Rg = \frac{v_e^2}{2}.$$

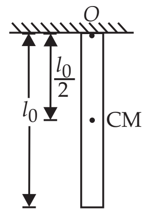
$$\left(\because\ g = \frac{GM}{R^2}\right)$$

$$\text{or} \quad v = \frac{v_e}{\sqrt{2}}$$

18. (d) : $B_0 = \dfrac{E_0}{c} = \dfrac{54}{3 \times 10^8} = 18 \times 10^{-8}$ T

$$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{m} = 1.5 \text{ cm}$$

19. (d) : Here the rod is oscillating about an end point *O*. Hence, moment of inertia of rod about the point of oscillation is



$$I = \frac{1}{3}ml_0^2$$

Moreover, length *l* of the pendulum = distance from the oscillation axis to centre of mass of rod = $l_0/2$

∴ Time period of oscillation

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{3}ml_0^2}{mg\left(\frac{l_0}{2}\right)}}\ ,\ 2\pi\sqrt{\frac{2l_0}{3g}}$$

20. (c) :

	P	Q
No. of nuclei, at $t = 0$	$4N_0$	N_0
Half- life	1 min	2 min
No. of nuclei after time t	N_P	N_Q

Let after *t* min the number of nuclei of *P* and *Q* are equal.

$$\therefore\ N_P = 4N_0\left(\frac{1}{2}\right)^{t/1} \text{ and } N_Q = N_0\left(\frac{1}{2}\right)^{t/2}$$

As $N_P = N_Q$

$$\therefore\ 4N_0\left(\frac{1}{2}\right)^{t/1} = N_0\left(\frac{1}{2}\right)^{t/2},\ \frac{4}{2^{t/1}} = \frac{1}{2^{t/2}},\ 4 = \frac{2^t}{2^{t/2}}$$

$4 = 2^{t/2}, 2^2 = 2^{t/2}$

$\frac{t}{2} = 2$ or $t = 4$ min

After 4 minutes, both *P* and *Q* have equal number of nuclei.

∴ Number of nuclei of *R*

$$= \left(4N_0 - \frac{N_0}{4}\right) + \left(N_0 - \frac{N_0}{4}\right)$$

$$= \frac{15N_0}{4} + \frac{3N_0}{4} = \frac{9N_0}{2}$$

21. (5.0) : Here, velocity of projection, $u = 5\sqrt{2}$ m s^{-1}

At the highest point velocity of projectile,

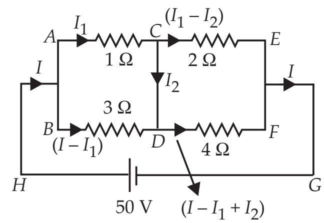
$v = u\cos\theta$

$$\therefore\ 5 = 5\sqrt{2}\cos\theta,\ \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

$$\text{Range} = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{(5\sqrt{2})^2\sin(2\times 45°)}{10} = \frac{25\times 2}{10} = 5\text{ m}$$

22. (2.0) :



Applying Kirchhoff's second law for the closed loop *ACDBA*, we get

$-I_1 + 3(I - I_1) = 0$

$-I_1 + 3I - 3I_1 = 0$

$3I - 4I_1 = 0$ ...(i)

Applying Kirchhoff's second law for the closed loop *CEFDC*

$-2(I_1 - I_2) + 4(I - I_1 + I_2) = 0$

$-2I_1 + 2I_2 + 4I - 4I_1 + 4I_2 = 0$

$4I - 6I_1 + 6I_2 = 0$

$2I - 3I_1 + 3I_2 = 0$ ...(ii)

Applying Kirchhoff's second law for the closed loop *AEGHA*

$-I_1 - 2(I_1 - I_2) + 50 = 0$

$-I_1 - 2I_1 + 2I_2 + 50 = 0$

$-3I_1 + 2I_2 + 50 = 0$ ...(iii)

From equation (i), we get $I = \frac{4I_1}{3}$

Substituting the value of *I* in equation (ii)

$$2\left(\frac{4I_1}{3}\right) - 3I_1 + 3I_2 = 0$$

$8I_1 - 9I_1 + 9I_2 = 0$

$-I_1 + 9I_2 = 0$ ...(iv)

Solving equation (iii) and (iv) we get

$I_2 = 2$ A.

Hence, current passing through wire *CD* is 2 A.

23. (1.0) : Since frequencies are in odd number ratio, the pipe has to be a closed pipe.

Ratio of 3 frequencies = 425 : 595 : 765 = 5 : 7 : 9

The frequency of 5th harmonic (*i.e.* 2nd overtone) is 425 Hz.

$$\therefore\ 425 = 5\upsilon \Rightarrow \upsilon = \frac{425}{5} = 85\text{ Hz}$$

where υ is the fundamental frequency of the pipe.

Fundamental frequency, $\upsilon = \frac{v}{4L}$

where *L* is the length of the pipe and *v* is the speed of sound in air.

$$\text{or, } L = \frac{v}{4\upsilon} = \frac{340}{4\times 85} = 1\text{ m.}$$

24. (3.0) : As $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

Apparent depth $= \dfrac{12}{(4/3)} = 9$ cm

The height through which image of fish is raised
$= 12 - 9 = 3$ cm.

25. (2.0) : de Broglie wavelength, $\lambda = \dfrac{h}{mv}$
According to given problem

$\frac{1}{2} m_\alpha (v_\alpha)^2 = \frac{1}{2} m_p (v_p)^2$

$\frac{1}{2} 4m_p (v_\alpha)^2 \quad \frac{1}{2} m_p (v_p)^2$ or $\frac{v_\alpha}{v_p} = \frac{1}{2}$

$\lambda_\alpha = \frac{h}{m_\alpha v_\alpha}; \quad \lambda_p = \frac{h}{m_p v_p}$

$\frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha v_\alpha}{m_p v_p} = 4 \times \frac{1}{2} = 2 \qquad \left(\because \frac{m_\alpha}{m_p} = 4\right)$

26. (b) : H_2O is a weak field ligand, hence $\Delta_o <$ pairing energy.
CFSE $= (-0.4x + 0.6y)\Delta_o$
where, x and y are no. of electrons occupying t_{2g} and e_g orbitals respectively.
For $[Fe(H_2O)_6]^{3+}$ complex ion,
$Fe^{3+}(3d^5) : t_{2g}^3 e_g^2 = -0.4 \times 3 + 0.6 \times 2 = 0.0$ or 0 Dq

27. (a)

28. (d) : Macromolecular colloids are quite stable and resemble true solution in many respects.

29. (d) : Ionization enthalpy increases on moving from left to right across the period as the size decreases and decreases on moving top to bottom in a group as the size increases. Ar has the maximum value of *I.E.*, since it is a noble gas. So, the correct order of increasing first ionization enthalpy ($\Delta_i H_1$) is Ba < Ca < Se < S < Ar.

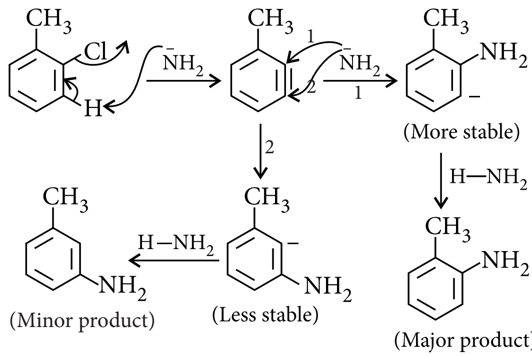
30. (a) : $Ti(SO_4)_2 + H_2O_2 + 2H_2O \longrightarrow \underset{\text{Yellow colour or Orange colour}}{H_2TiO_4} + 2H_2SO_4$

31. (d) : Order of reaction $= \frac{1}{3} + \frac{2}{3} = 1.$

Unit of rate constant for 1st order reaction = time^{-1}

32. (c) : The groups having $+I$ effect decrease the stability while groups having $-I$ effect increase the stability of carbanions. Benzyl carbanion is stabilized due to resonance. Also, out of 2° and 3° carbanions, 2° carbanions are more stable, thus the decreasing order of stability is
$\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$

33. (a) :

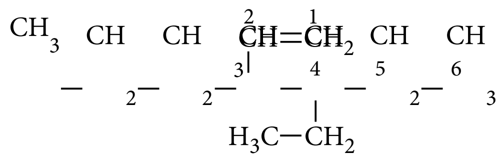


34. (c) : The two strands in a DNA molecule are not exactly similar but are complementary.

35. (d) : In photoelectric effect, kinetic energy and hence velocity is directly proportional to frequency of the incident light and independent of the intensity of light.

36. (a)

37. (b) :

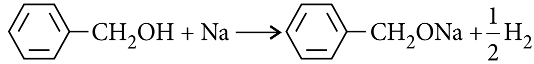


4-Ethyl-3-propylhex-1-ene

38. (c) : No. of atoms of A from corners of unit cell = 7/8
No. of atoms of B from faces of unit cell = 3
Thus, $A : B = 7/8 : 3$ or $7 : 24$. Thus, formula is A_7B_{24}.

39. (a) : Borax is used as a buffer as it contains weak acid and its salt with strong base.
$Na_2B_4O_7 + 7H_2O \longrightarrow 2Na[B(OH)_4] + 2H_3BO_3$

40. (a) : The compound has no phenolic group, but has an alcoholic group (—OH).



41. (c) : Reaction of methylamine and phosgene to produce MIC (methyl isocyanate) is not an example of green chemistry.

42. (a) : Alkali metal oxides are very much basic. Down the group in the alkaline earth metal series, ionisation potential decreases.

43. (c) : The concept is based on thermodynamically and kinetically controlled reaction.

When the temperature is lower, reaction is irreversible and kinetically controlled. So, due to more polarity of $>C=O$ bond, addition takes place across $>C=O$.

When the temperature is higher, the reaction is reversible and hence thermodynamically controlled. $>C=O$ is thermodynamically more stable than $>C=C<$ hence, addition takes place across $>C=C<$.

44. (b) : $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$

$$Q_c = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

Given, $[NH_3] = \frac{8.13}{20} M = 0.4065\ M$;

$[N_2] = \frac{1.57}{20} M = 0.0785\ M$

$[H_2] = \frac{1.92}{20} M = 0.096\ M$

$$Q_c = \frac{(0.4065\ M)^2}{(0.0785\ M)(0.096\ M)^3} = 2.379 \times 10^3\ M^{-2}$$

$Q_c \neq K_c$, so the reaction is not in equilibrium. $Q_c > K_c$, it indicates that the reaction will proceed in the direction of reactants.

45. (b) : According to Kohlrausch's law, the molar conductivity at infinite dilution (Λ°) for a weak electrolyte, CH_3COOH is,

$\Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$

So, for calculating the value of $\Lambda^\circ_{CH_3COOH}$, value of Λ°_{NaCl}, should also be known.

46. (6.47) : Number of moles of $FeSO_4$

$$= \frac{20\ g}{152\ g\ mol^{-1}} = 0.132\ mol$$

Ionic equation for oxidation of Fe^{2+} ions to Fe^{3+} ions from $K_2Cr_2O_7$ in acidic medium is

$Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \rightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$

6 moles of $Fe^{2+} \equiv$ 1 mol of $K_2Cr_2O_7$

$\therefore$ 0.132 moles of $Fe^{2+} \equiv \frac{1}{6} \times 0.132 = 0.022$ mol of $K_2Cr_2O_7$

$\Rightarrow$ Mass of $K_2Cr_2O_7$ in 0.022 mol

$= 0.022\ mol \times 294\ g\ mol^{-1} = 6.468\ g \approx 6.47g$

47. (36) : Molecular weight of metal sulphate $= 90 \times 2 = 180$

$$\text{Equivalent mass of metal} = \frac{\text{Mass of element}}{\text{Mass of oxygen}} \times 8$$

$$= \frac{60}{40} \times 8 = 12$$

Equivalent mass of metal sulphate = 12 + 48 = 60

$$\text{Valency} = \frac{180}{60} = 3$$

$\Rightarrow$ At. wt. of metal $= 12 \times 3 = 36$

48. (10) : According to Raoult's law, relative lowering of vapour pressure,

$$\frac{p^\circ_A - p_s}{p^\circ_A} = x_B \quad \text{...(i)}$$

$$x_B = \frac{n_B}{n_B + n_A} = \frac{W_B/M_B}{\frac{W_B}{M_B} + \frac{W_A}{M_A}} \quad \text{...(ii)}$$

Given vapour pressure is reduced to 80% when non-volatile solute is dissolved in octane *i.e.*, if $p^\circ_A = 1$ atm, then $p_s = 0.8$ atm; $p^\circ_A - p_s = 0.2$ atm; $M_A(C_8H_{18}) = 114\ g\ mol^{-1}$; $W_A = 114$ g; $M_B = 40\ g\ mol^{-1}$; $W_B = ?$

From eq. (i) and (ii),

$$\frac{0.2}{1} = \frac{W_B/40}{\frac{W_B}{40} + \frac{114}{114}} = \frac{W_B/40}{\frac{W_B}{40} + 1} \Rightarrow 0.2 = \frac{W_B}{W_B + 40}$$

$0.2W_B + 8 = W_B \Rightarrow W_B = 10$ g

49. (25.37) : Suppose volume of 200 mg of air at 17°C = V mL

As pressure remains constant (being an open vessel), applying Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}, \text{ i.e., } \frac{V}{290} = \frac{V_2}{390} \text{ or } V_2 = 1.34\ V$$

$\therefore$ Volume of air expelled $= 1.34\ V - V = 0.34\ V$

Mass of 1.34 V air at 117°C = 200 mg

Mass of 0.34 V air at 117°C $= \frac{200}{1.34} \times 0.34$ mg

$\therefore$ Mass % of air expelled

$$\frac{200 \times 0.34}{1.34} \times \frac{1}{200} \times 100 = 25.37\%$$

50. (3.58) : The burning of methane may be expressed as

$CH_{4(g)} + 2O_{2(g)} \rightarrow CO_{2(g)} + 2H_2O_{(g)}$

$\Delta_f H^\circ = [\Delta_f H^\circ (CO_2) + 2\Delta_f H^\circ (H_2O)] - [\Delta_f H^\circ (CH_4) + 2\Delta_f H^\circ (O_2)]$

$= [-394.8 + 2 \times (-241.6)] - [-76.2 + 2 \times (0)]$

$= -801.8$ kJ

1 mole or 22.4 L of CH_4 evolve heat = 801.8 kJ

1 m^3 or 1000 L of CH_4 evolve heat $= \dfrac{801.8 \times 1000}{22.4}$

$= 35794.6$ kJ

$\approx 3.58 \times 10^4$ kJ

51. (d) : The given relation may be written in set-builder form as

$R = \{(a,b) : a - b \text{ divides } n \text{ and } a, b \in Z\}$

As $a - a = 0$ and 0 divides n $\therefore$ $(a, a) \in R$

$\therefore$ R is reflexive.

Let $a, b \in Z$ such that $(a, b) \in R$

Then $(a, b) \in R \Rightarrow a - b$ divides n.

$a - b = nk$ for some integer $k \Rightarrow b - a = n(-k)$

$\therefore$ $(a, b) \in R \Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

Now, $(a,b), (b,c) \in R$

Now, $a - b = nc_1$ and $b - c = nc_2$ for some integers c_1 and c_2.

$\therefore$ $(a - b) + (b - c) = n(c_1 + c_2)$

$\Rightarrow$ $a - c = nk$, where $k = c_1 + c_2$, an integer.

$\Rightarrow$ $(a, c) \in R$.

$\therefore$ $(a,b), (b,c) \in R \Rightarrow (a,c) \in R$

$\therefore$ R is transitive and hence R is an equivalence relation.

52. (a) : The system will have a non-zero solution, if

$$\Delta \equiv \begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a^3 & 3a^2+3a+1 & 3(a+1)^2+3(a+1)+1 \\ a & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

(by $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$)

$\Rightarrow$ $3a^2 + 3a + 1 - \{3(a + 1)^2 + 3(a + 1)+1\} = 0$

(Expanding along R_3)

$\Rightarrow$ $-6(a + 1) = 0 \Rightarrow a = -1$

53. (a) : Given, $f(x) = 5 + 36x + 3x^2 - 2x^3$

$\Rightarrow$ $f'(x) = 36 + 6x - 6x^2 = 6(6 + x - x^2)$

For increasing or decreasing, $f'(x) = 0$

$\Rightarrow$ $6 + x - x^2 = 0 \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow$ $(x - 3)(x + 2) = 0 \Rightarrow$ $x = 3, -2$

$-\infty < x < -2, f'(x) = (-ve)(-ve) = (+ve)$, Increasing

$-2 < x < 3, f'(x) = (-ve)(+ve) = (-ve)$, Decreasing

$3 < x < \infty, f'(x) = (+ve)(+ve) = (+ve)$, Increasing

$\therefore$ The interval in which $f(x)$ is decreasing is $(-2, 3)$.

54. (c) : We have, $7^{300} = (7^2)^{150} = (50 - 1)^{150}$

$= {}^{150}C_0 (50)^{150} (-1)^0 + {}^{150}C_1 (50)^{149} (-1)^1 + \ldots + {}^{150}C_{150} (50)^0 (-1)^{150}$

Thus the last digits of 7^{300} are ${}^{150}C_{150} \cdot 1 \cdot 1$ *i.e.*, 1.

55. (c) : $\displaystyle\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right] dx$

$\displaystyle = \int_0^1 \frac{d}{dx}(2\tan^{-1} x)\, dx$

$\displaystyle = \int_0^1 \frac{2}{1+x^2}\, dx = 2\left[\tan^{-1} x\right]_0^1 = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

56. (d) : For $f(x)$ to be continuous at $x = 0$, we have $\displaystyle\lim_{x \to 0} f(x) = f(0) = 12(\log 4)^3$ ...(i)

Now,

$$\lim_{x\to 0} f(x) = \lim_{x \to 0}\left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{p}\right)}{\left(\sin\frac{x}{p}\right)} \cdot \frac{px^2}{\log\left(1 + \frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 \cdot 1 \cdot p \cdot \lim_{x \to 0}\left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \ldots}\right)$$

$= 3p(\log 4)^3$ ...(ii)

$\therefore$ On comparing (i) and (ii), we get $p = 4$.

57. (c) : Since, points (3, 3), $(h, 0)$ and $(0, k)$ are collinear, so one point will lie on the line joining the other two points

i.e., $y - 0 = \dfrac{k-0}{0-h}(x - h)$

$\therefore$ $3 = -\dfrac{k}{h}(3 - h)$ ($\because$ (3, 3) lies on the line)

$\Rightarrow$ $\dfrac{3}{h} + \dfrac{3}{k} = 1 \Rightarrow \dfrac{1}{h} + \dfrac{1}{k} = \dfrac{1}{3}$

Comparing with $\dfrac{a}{h} + \dfrac{b}{k} = \dfrac{1}{3}$, we get $a = 1, b = 1$

58. (b) : Given, $iz^3 + z^2 - z + i = 0$

Dividing both side by i and using $\dfrac{1}{i} = -i$, we have

$z^3 - iz^2 + iz + 1 = 0$

$\Rightarrow$ $z^2(z - i) + i(z - i) = 0$ ($\because i^2 = -1$)

$\Rightarrow\ (z-i)(z^2+i)=0 \ \therefore\ z=i$ or $z^2=-i$

$\therefore\ |z|=|i|=1$ and $|z^2|=|z|^2=|-1|=1\ \therefore\ |z|=1$

59. (c) : $\dfrac{1+\sin A-\cos A}{1+\sin A+\cos A}$

$$=\frac{2\sin^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=\frac{2\sin\frac{A}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}=\tan\frac{A}{2}$$

60. (b) : Let four arithmetic means are A_1, A_2, A_3 and A_4. So, $3, A_1, A_2, A_3, A_4, 23$

$\Rightarrow\ T_6=23=a+5d\ \Rightarrow\ d=4$

Thus $A_1=3+4=7, A_2=7+4=11,$

$A_3=11+4=15, A_4=15+4=19$

61. (d) : $\lim\limits_{n\to\infty}\dfrac{n(n+1)}{2}\cdot\dfrac{n^2(n+1)^2}{4}\cdot\dfrac{36}{n^2(n+1)^2(2n+1)^2}$

$$=\lim_{n\to\infty}\frac{9}{2}\frac{\left(1+\frac{1}{n}\right)}{\left(2+\frac{1}{n}\right)^2}=\frac{9}{8}$$

62. (b) : If the planes $x-cy-bz=0$, $-cx+y-az=0$, $-bx-ay+z=0$ pass through a line, then determinant formed by coefficients of unknowns is equal to zero.

$$\Rightarrow\begin{vmatrix}1 & -c & -b\\ -c & 1 & -a\\ -b & -a & 1\end{vmatrix}=0$$

$\Rightarrow\ 1(1-a^2)+c(-c-ab)-b(ca+b)=0$

$\Rightarrow\ 1-a^2-c^2-abc-abc-b^2=0$

$\Rightarrow\ a^2+b^2+c^2+2abc-1=0$

$\Rightarrow\ a^2+b^2+c^2+2abc=1$

63. (d) : The given differential equation is

$$\frac{dy}{dx}=\frac{1+y^2}{1+x^2}\ \Rightarrow\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$$

$$\Rightarrow\int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$$

$$\Rightarrow\ \tan^{-1}y=\tan^{-1}x+\tan^{-1}c$$

$$\Rightarrow\tan^{-1}\left(\frac{y-x}{1+xy}\right)=\tan^{-1}c$$

$\Rightarrow\ \dfrac{y-x}{1+xy}=c\ \Rightarrow\ y-x=c(1+xy)$, which is the required solution of the given differential equation.

64. (b) : $\bar{x}=72$

(Number of boys)$n_1=70$

(Number of girls)$n_2=30$

$\bar{x}_1=75$; $\bar{x}_2=?$

$$\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}\ \Rightarrow\ 72=\frac{70\times75+\bar{x}_2\times30}{70+30}$$

$\Rightarrow\bar{x}_2=65.$

65. (c)

66. (d) : Let E_1 = Event of choosing purse I

E_2 = Event of choosing purse II

A = Event that the coin drawn is of copper.

$\therefore\ P(E_1)=\dfrac{1}{2}=P(E_2)$

$\therefore\ P(A|E_1)=\dfrac{4}{7},\ P(A|E_2)=\dfrac{3}{4}$

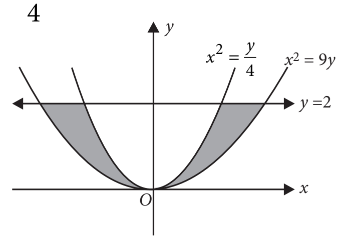
By theorem of total probability,

$P(A)=P(E_1)\,P(A\mid E_1)+P(E_2)P(A\mid E_2)$

$$=\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{4}=\frac{37}{56}$$

67. (d) : Given, $y=4x^2$

$\Rightarrow\ x^2=\dfrac{y}{4}$ ... (i)



$x^2=9y$ ...(ii)

and $y=2$ ...(iii)

$\therefore$ Area bounded by the above three curves

$$=2\int_0^2\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy=2\int_0^2\frac{5}{2}\sqrt{y}\,dy=2\times\frac{5}{2}\left[\frac{2y^{3/2}}{3}\right]_0^2$$

$$=\frac{10}{3}[2\sqrt{2}-0]=\frac{20\sqrt{2}}{3}\ \text{sq. unit}$$

68. (a) : Centres : $C_1(1, 3), C_2(4, -1)$,

Radii : $r_1=r, r_2=\sqrt{16+1-8}=3$

$C_1C_2 = \sqrt{9+16} = 5, |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow \ |r - 3| < 5 < r + 3$

$\therefore \ r > 2, |r - 3| < 5 \Rightarrow r - 3 < 5, r < 8$

$\therefore \ 2 < r < 8.$

69. (b) : Given, α, β are the roots of equation

$x^2 - (a - 2)x - a - 1 = 0$

$\therefore \ \alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6 = a^2 - 2a + 1 + 5$

$= (a - 1)^2 + 5 \geq 5$

$\therefore \ \alpha^2 + \beta^2$ is least if $(a - 1)^2 = 0 \ \Rightarrow \ a = 1.$

70. (b) : We have $\tan\theta = \dfrac{x\sin\phi}{1 - x\cos\phi}$

$\Rightarrow x\sin\phi = \tan\theta - x\cos\phi\tan\theta$

$$\Rightarrow x = \frac{\tan\theta}{\sin\phi + \cos\phi\tan\theta}$$

$$= \frac{\sin\theta}{\cos\theta\sin\phi + \cos\phi\sin\theta} = \frac{\sin\theta}{\sin(\theta+\phi)}$$

Similarly, $y = \dfrac{\sin\phi}{\sin(\theta+\phi)}$; $\therefore \dfrac{x}{y} = \dfrac{\sin\theta}{\sin\phi}$.

71. (8) : $\text{adj}\, P = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 7 & 3 \end{bmatrix}$

$|\text{adj}\, P| = 4 \ \Rightarrow \ |P|^2 = 4 \ \Rightarrow \ |P| = \pm 2$

$\therefore$ Required sum $= (2)^2 + (-2)^2 = 8$

72. (100) : $x^2 + 1 = (x + i)(x - i)$

$b = 1, a = c$

No. of ways of choosing $a, b, c = 10$

$\therefore \ 10K = 10 \times 10 = 100$

73. (12) : Let the series be $a, ax, ax^2, ax^3, ...$

Also, $\dfrac{T_4}{T_2} = \dfrac{ax^3}{ax} = \dfrac{1}{16} \ \Rightarrow \ x^2 = \dfrac{1}{16} \ \Rightarrow \ x = \pm\dfrac{1}{4}$

But since it is a decreasing G.P. $\therefore \ x = \dfrac{1}{4}$

Also, $\dfrac{T_3}{T_2^2} = \dfrac{ax^2}{(ax)^2} = \dfrac{1}{9} \ \Rightarrow \ \dfrac{1}{a} = \dfrac{1}{9} \ \Rightarrow \ a = 9$

Now, $S_\infty = \dfrac{a}{1-r} = \dfrac{9}{1-\frac{1}{4}} = \dfrac{9\times 4}{3} = 12$

74. (0.77) : Let equation of new plane

$2x - 2y + z - 3 + \lambda z = 0$ ...(i)

Given, point (3, 1, 1) lie on plane (i) $\therefore \lambda = -2$

Hence equation of new plane is $2x - 2y - z = 3$

$\cos\alpha = \dfrac{4+4-1}{3\cdot 3} = \dfrac{7}{9} = 0.77$

75. (12) : Let $\vec{a} = 10\hat{i} + 3\hat{j}, \vec{b} = 12\hat{i} - 5\hat{j}$

and $\vec{c} = \lambda\hat{i} + 11\hat{j}$

Since $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear. Therefore, area of the triangle formed by them is zero.

i.e., $\begin{vmatrix} 10 & 3 & 1 \\ 12 & -5 & 1 \\ \lambda & 11 & 1 \end{vmatrix} = 0$

$\Rightarrow \ 10(-5 - 11) - 3(12 - \lambda) + 1(132 + 5\lambda) = 0$

$\Rightarrow \ 8\lambda - 64 = 0 \ \Rightarrow \ \lambda = 8$

$\therefore \ \dfrac{3}{2}\lambda = \dfrac{3}{2} \times 8 = 12$