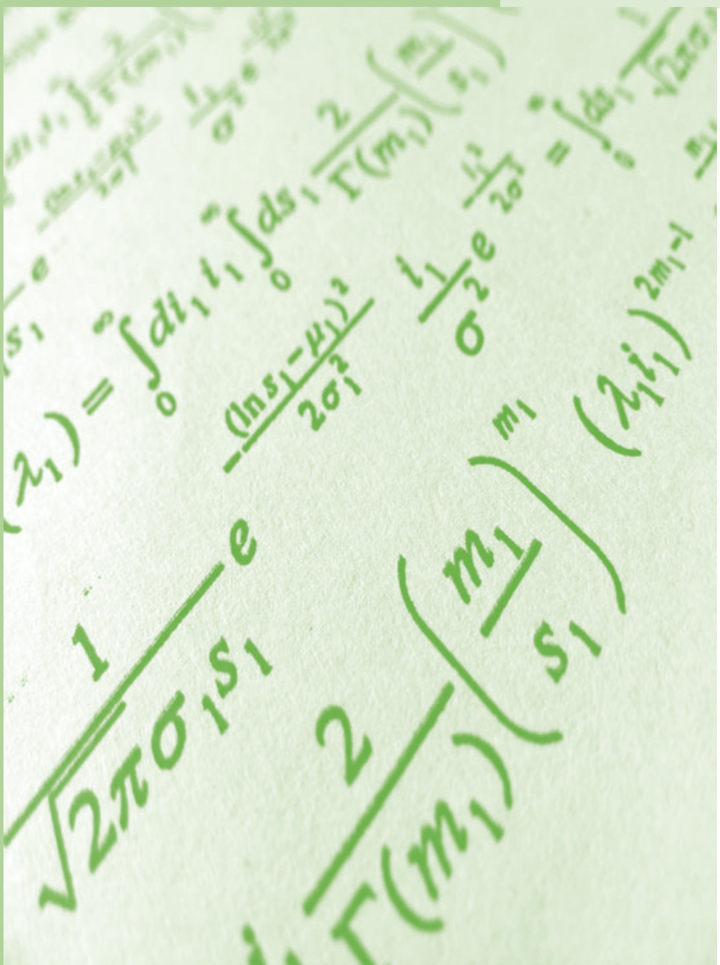
Chapter

15

Coordinate Geometry



REMEMBER

Before beginning this chapter, you should be able to:

- Study coordinates of a point and conversion of signs
- Study about points on the plane, distance between the points

KEY IDEAS

After completing this chapter, you would be able to:

- Study distance between a point and the axes and apply the distance formula in solving word problems
- Learn signs of coordinates of a point in four quadrants
- Understand inclination and slope of a straight line
- Obtain equation of a line parallel or perpendicular to the given line
- Find out midpoint, centroid, median and altitude of a triangle
- Calculate areas of triangle and quadrilateral

INTRODUCTION

Let $X'OX$ and YOY' be two mutually perpendicular lines intersecting at the point O in a plane.

These two lines are called reference lines or coordinate axes. The horizontal reference line $X'OX$ is called X-axis and the vertical reference line YOY' is called Y-axis.

The point of intersection of these two axes, i.e., O is called the origin. The plane containing the coordinate axes is called coordinate plane or XY-plane.

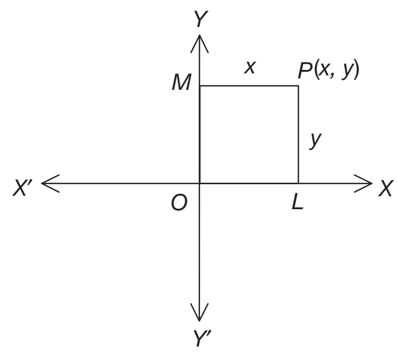


Figure 15.1

COORDINATES OF A POINT

Let P be a point in the XY-plane. Draw perpendiculars PL and PM to X-axis and Y-axis respectively.

Let $PL = y$ and $PM = x$. Then, the point P is taken as (x, y). Here x and y are called the rectangular Cartesian coordinates or simply coordinates of the point P. x is called x-coordinate or abscissa and y is called y-coordinate or ordinate of the point P. $P(x, y)$ is x units away from Y-axis and y units away from X-axis.

Convention of Signs

1. Towards the right side of the Y-axis, x-coordinate of any point on the graph paper is taken positive and towards the left side of the Y-axis, x-coordinate is taken negative.
2. Above the X-axis, the y-coordinate of any point on the graph paper is taken positive and below the X-axis, y-coordinate is taken negative.

If (x, y) is a point in the plane and Q_1, Q_2, Q_3, Q_4 are the four quadrants of rectangular coordinate system, then

1. If $x > 0$ and $y > 0$, then $(x, y) \in Q_1$.
2. If $x < 0$ and $y > 0$, then $(x, y) \in Q_2$.
3. If $x < 0$ and $y < 0$, then $(x, y) \in Q_3$.
4. If $x > 0$ and $y < 0$, then $(x, y) \in Q_4$.

EXAMPLE 15.1

If $x < 0$ and $y > 0$, then $(-x, y)$ lies in which quadrant?

SOLUTION

$x < 0 \quad \Rightarrow \quad -x > 0$

$\therefore$ The point $(-x, y)$ lies in the first quadrant, i.e., Q_1.

EXAMPLE 15.2

If $(a, b) \in Q_3$, then $(-a, -b)$ belongs to which quadrant?

SOLUTION

Given $(a, b) \in Q_3$, $\Rightarrow$ $a < 0$, $b < 0$ then $-a$ is positive and $-b$ is also positive therefore $(-a, -b) \in Q_1$.

EXAMPLE 15.3

Plot the points $A(2, 3)$, $B(-1, 2)$, $C(-3, -2)$, and $D(4, -2)$ in the XY-plane.

SOLUTION

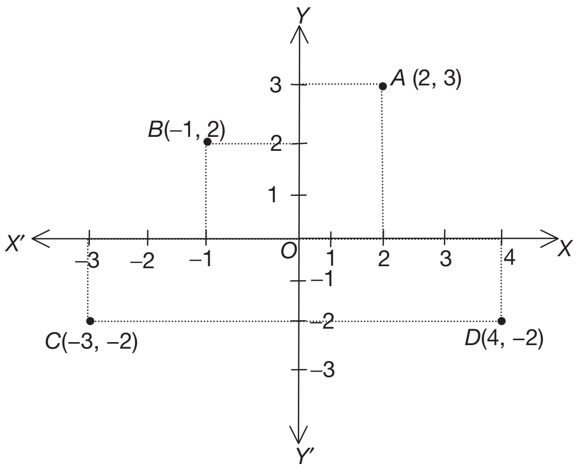


Figure 15.2

POINTS ON THE PLANE

Point on X-axis and Y-axis

Let P be a point on X-axis, so that its distance from X-axis is zero. Hence, the point P can be taken as $(x, 0)$.

Let P' be a point on Y-axis, so that its distance from Y-axis is zero. Hence, the point P' can be taken as $(0, y)$.

Distance Between Two Points

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$. Draw perpendiculars AL and BM from A and B to X-axis, AN is the perpendicular drawn from A on to BM.

From right triangle ABN, $AB = \sqrt{AN^2 + BN^2}$.

Here, $AN = x_2 - x_1$, and $BN = y_2 - y_1$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Hence, the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ units.}$$

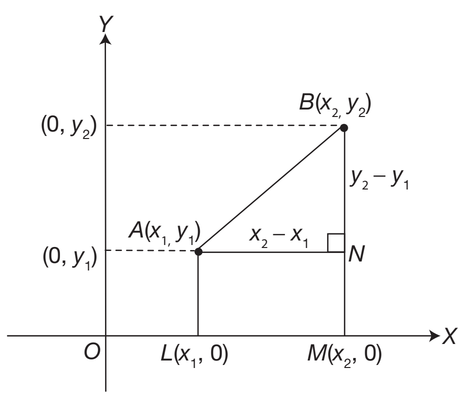


Figure 15.3

Note The distance of a point $A(x_1, y_1)$ from origin $O(0, 0)$ is $OA = \sqrt{x_1^2 + y_1^2}$.

EXAMPLE 15.4

Find the distance between the points (3, −5) and (5, −1).

SOLUTION

Let the given points be $A(3, -5)$ and $B(5, -1)$.

$$AB = \sqrt{(5-3)^2 + (-1-(-5))^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \text{ units.}$$

EXAMPLE 15.5

Find a if the distance between the points $P(11, -2)$ and $Q(a, 1)$ is 5 units.

SOLUTION

Given, $PQ = 5$

$$\Rightarrow \quad \sqrt{(a-11)^2 + (1-(-2))^2} = 5$$

Taking square on both sides, we get

$$(a-11)^2 = 25 - 9 = 16$$
$$a = 11 = \sqrt{16}$$
$$a - 11 = \pm 4$$
$$a = 15 \text{ or } 7.$$

EXAMPLE 15.6

Find the coordinates of a point on Y-axis which is equidistant from the points (13, 2) and (12, −3).

SOLUTION

Let $P(0, y)$ be the required point and the given points be $A(12,-3)$ and $B(13, 2)$.

Then $PA = PB$ (given)

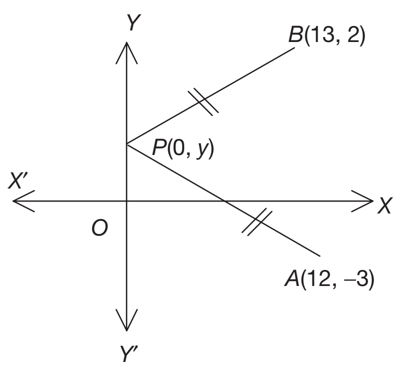


Figure 15.4

$$\sqrt{(12-0)^2 + (-3-y)^2} = \sqrt{(13-0)^2 + (2-y)^2} \quad \Rightarrow \quad \sqrt{144 + (y+3)^2} = \sqrt{169 + (2-y)^2}$$

Taking square on both sides, we get

$$144 + 9 + y^2 + 6y = 169 + 4 + y^2 - 4y$$
$$\Rightarrow \quad 10y = 20 \quad \Rightarrow \quad y = 2.$$

$\therefore$ The required point on Y-axis is (0, 2).

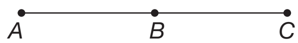
Applications of Distance Formula

Collinearity of Three Points Let A, B and C be three given points. The distances AB, BC and CA can be calculated using distance formula.

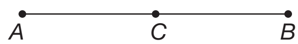
If the sum of any two of these distances is found to be equal to the third distance, then the points A, B and C will be collinear.

Notes

1. If $AB + BC = AC$, then the points A, B and C are collinear.



2. If $AC + CB = AB$, then the points A, C and B are collinear.



3. $BA + AC = BC$, then the points B, A and C are collinear.

By notes (1), (2) and (3), we can find the position of points in collinearity.

EXAMPLE 15.7

Show that the points $A(2, 3)$, $B(3, 4)$ and $C(4, 5)$ are collinear.

SOLUTION

Given, $A = (2, 3)$, $B = (3, 4)$ and $C = (4, 5)$

$$AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2} \text{ units.}$$
$$BC = \sqrt{(4-3)^2 + (5-4)^2} = \sqrt{2} \text{ units.}$$
$$AC = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

Now, $AB + BC = \sqrt{2} + \sqrt{2} = 2\sqrt{2} = AC$

That is, $AB + BC = AC$.

Hence, the points A, B and C are collinear.

EXAMPLE 15.8

Show that the points (2, 4), (6, 8) and (2, 8) form an isosceles right triangle when joined.

SOLUTION

Let $A(2, 4)$, $B(6, 8)$ and $C(2, 8)$ be the given points.

$$AB = \sqrt{(6-2)^2 + (8-4)^2} = \sqrt{32} \text{ units.}$$

$$BC = \sqrt{(2-6)^2 + (8-8)^2} = \sqrt{16} = 4 \text{ units.}$$

$$AC = \sqrt{(2-2)^2 + (8-4)^2} = \sqrt{16} = 4 \text{ units.}$$

Clearly, $BC^2 + AC^2 = AB^2$.

$\Rightarrow \quad \angle C = 90°$

Also, $BC = AC$.

Hence, the given points form the vertices of a isosceles right triangle.

EXAMPLE 15.9

Show that the points (1, –1), (–1, 1) and $(\sqrt{3}, \sqrt{3})$ when joined, form an equilateral triangle.

SOLUTION

Let $A(1, -1)$, $B(-1, 1)$ and $C(\sqrt{3}, \sqrt{3})$ be the given points.

Then, $AB = \sqrt{(-1-1)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8}$ units.

$$BC = \sqrt{(\sqrt{3}-(-1))^2 + (\sqrt{3}-1)^2} = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}$$

$$= \sqrt{2(3+1)} \; [\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)] = \sqrt{8} \text{ units.}$$

$$CA = \sqrt{(1-\sqrt{3})^2 + (-1-\sqrt{3})^2} = \sqrt{(1-\sqrt{3})^2 + (1+\sqrt{3})^2} = \sqrt{8} \text{ units.}$$

$\therefore \quad AB = BC = CA = \sqrt{8}$

Hence, the points (1, –1), (–1, 1) and $(\sqrt{3}, \sqrt{3})$ when joined, form an equilateral triangle.

EXAMPLE 15.10

Show that the points (2, 0), (–6, –2), (–4, –4) and (4, –2) form a parallelogram.

SOLUTION

Let the given points be $A(2, 0)$, $B(-6, -2)$, $C(-4, -4)$ and $D(4, -2)$

Then, $AB = \sqrt{(-6-2)^2 + (-2-0)^2} = \sqrt{68}$ units.

$$BC = \sqrt{(-4+6)^2 + (-4+2)^2} = \sqrt{8} \text{ units.}$$

$$CD = \sqrt{(4-4)^2 + (-4+2)^2} = \sqrt{68} \text{ units.}$$

$$DA = \sqrt{(4-2)^2 + (-2-0)^2} = \sqrt{8} \text{ units.}$$

$$AC = \sqrt{(-4-2)^2 + (-4-0)^2} = \sqrt{52} \text{ units.}$$

$$BD = \sqrt{(4+6)^2 + (-2+2)^2} = \sqrt{10} \text{ units.}$$

Clearly,

$AB = CD$, $BC = DA$ and $AC \neq BD$.

That is, the opposite sides of the quadrilateral are equal and diagonals are not equal.

Hence, the given points form a parallelogram.

EXAMPLE 15.11

The vertices of a ΔABC are $A(1, 2)$, $B(3, -4)$ and $C(5, -6)$. Find its circum-centre and circum-radius.

SOLUTION

Let $S(x, y)$ be the circum-centre of ΔABC.

$\therefore \quad SA^2 = SB^2 = SC^2.$

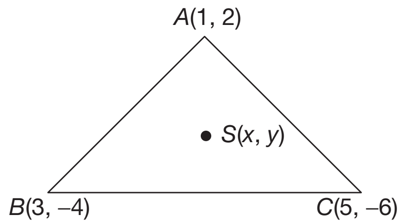


Figure 15.5

Consider

$$SA^2 = SB^2.$$

$\Rightarrow \quad (x - 1)^2 + (y - 2)^2 = (x - 3)^2 + (y + 4)^2.$

$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 + 8y + 16$

$-2x - 4y + 1 + 4 = -6x + 9 + 8y + 16$

$4x - 12y - 20 = 0$

$$x - 3y = 5 \tag{1}$$

$SB^2 = SC^2.$

$\Rightarrow \quad (x - 3)^2 + (y + 4)^2 = (x - 5)^2 + (y + 6)^2$

$$x - y = 9 \tag{2}$$

Solving Eqs. (1) and (2), we have

$x = 11$ and $y = 2$.

$\therefore \quad$ The required circum-centre of ΔABC is $(11, 2)$.

Circum-radius $= SA = \sqrt{(11-1)^2 + (2-2)^2} = 10$ units.

EXAMPLE 15.12

Find the area of the circle whose centre is $(-3, 2)$ and $(2, 5)$ is a point on the circle.

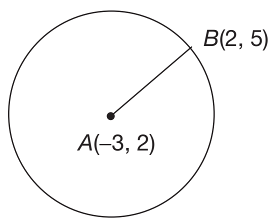


Figure 15.6

SOLUTION

Let the centre of the circle be $A(-3, 2)$ and point of the circumference be $B(2, 5)$

Radius of the circle $= AB = \sqrt{(2+3)^2 + (5-2)^2} = \sqrt{25+9}$

$r = \sqrt{34}$ units.

$\therefore \quad$ The area of circle $= \pi r^2$

$= \pi(\sqrt{34})^2 = 34\pi$ sq. units.

EXAMPLE 15.13

Find the area of a square whose one pair of the opposite vertices are $(3, 4)$ and $(5, 6)$.

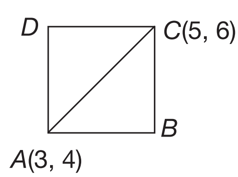


Figure 15.7

SOLUTION

Let the given vertices be $A(3, 4)$ and $C(5, 6)$.

Length of $AC = \sqrt{(3-5)^2 + (4-6)^2} = \sqrt{8}$ units.

Area of the square $= \dfrac{AC^2}{2} = \dfrac{(\sqrt{8})^2}{2} = 4$ sq. units.

EXAMPLE 15.14

Find the ortho-centre of the ΔABC formed by the vertices $A(2, 2)$, $B(6, 3)$ and $C(4, 11)$.

SOLUTION

The given vertices of ΔABC are $A(2, 2)$, $B(6, 3)$ and $C(4, 11)$.

$$\text{Length of } AB = \sqrt{(6-2)^2 + (3-2)^2} = \sqrt{17} \text{ units.}$$

$$\text{Length of } BC = \sqrt{(6-4)^2 + (3-11)^2} = \sqrt{68} \text{ units.}$$

$$\text{Length of } AC = \sqrt{(4-2)^2 + (11-2)^2} = \sqrt{85} \text{ units.}$$

Clearly, $AC^2 = AB^2 + BC^2$

ABC is a right triangle, right angle at B.

Hence, ortho-centre is the vertex containing right angle, i.e., $B(6, 3)$.

STRAIGHT LINES

Inclination of a Line

The angle made by a straight line with positive direction of X-axis in the anti-clockwise direction is called its inclination.

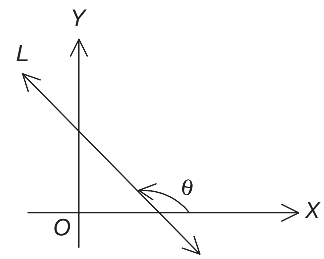


Figure 15.8

Slope or Gradient of a Line

If θ is the inclination of a line L, then its slope is denoted by m and is given by $m = \tan\theta$.

Example: The inclination of the line l in the Fig. 15.9 is 45°.

$\therefore$ The slope of the line is $m = \tan 45° = 1$.

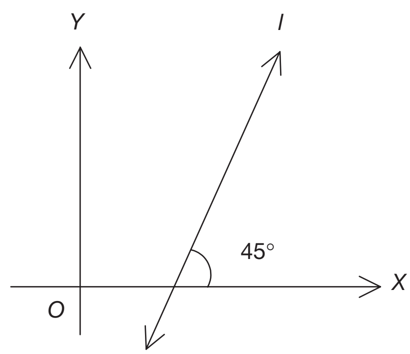


Figure 15.9

Example: The line L in the following figure makes an angle of 45° in clockwise direction with X-axis. So, the inclination of the line L is $180° - 45° = 135°$.

$\therefore$ The slope of the line L is $m = \tan 135° = -1$.

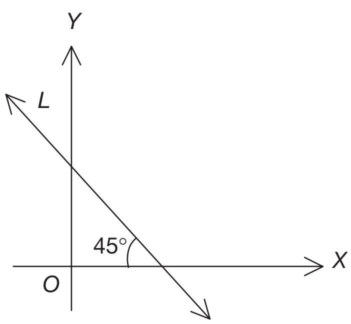


Figure 15.10

Some Results on the Slope of a Line

1. The slope of a horizontal line is zero. Hence,
   (i) Slope of X-axis is zero.
   (ii) Slope of any line parallel to X-axis is also zero.
2. The slope of a vertical line is not defined. Hence,
   (i) Slope of Y-axis is undefined
   (ii) Slope of any line parallel to Y-axis is also undefined.

Theorem 1

Two non-vertical lines are parallel if and only if their slopes are equal.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2 respectively.

If θ_1 and θ_2 are the inclinations of the lines, L_1 and L_2 respectively, then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$

Now, since $L_1 \parallel L_2$. Then, $\theta_1 = \theta_2$. ($\because$ They form a pair of corresponding angles)

$\Rightarrow \quad \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad m_1 = m_2$.

Conversely, let $m_1 = m_2$

$\Rightarrow \quad \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad \theta_1 = \theta_2$.

$\Rightarrow \quad L_1 \parallel L_2$ ($\because$ θ_1 and θ_2 form a pair of corresponding angles)

Hence, two non-vertical lines are parallel if and only if their slopes are equal.

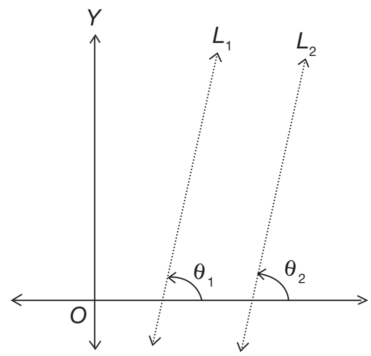


Figure 15.11

Theorem 2

Two non-vertical lines are perpendicular to each other if and only if the product of their slopes is -1.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes, m_1 and m_2.

If θ_1 and θ_2 are the inclinations of the lines L_1 and L_2 respectively, then

$m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$.

If $L_1 \perp L_2$, then

$\theta_2 = 90° + \theta_1$ ($\because$ The exterior angle of a triangle is equal to the sum of two opposite interior angles)

$\Rightarrow \quad \tan\theta_2 = \tan(90° + \theta_1)$

$\Rightarrow \quad \tan\theta_2 = -\cot\theta_1$

$\Rightarrow \quad \tan\theta_2 = \dfrac{-1}{\tan\theta_1} [\because \ \theta_1 \neq 0]$

$\Rightarrow \quad \tan\theta_1 \cdot \tan\theta_2 = -1$

$\therefore \quad m_1 m_2 = -1$.

Conversely, let $m_1 m_2 = -1$

$\Rightarrow \quad \tan\theta_1 \tan\theta_2 = -1$

$\Rightarrow \quad \tan\theta_2 = \dfrac{-1}{\tan\theta_1} [\because \ \theta_1 \neq 0]$

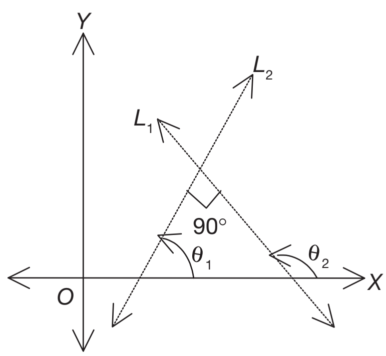


Figure 15.12

$\Rightarrow \quad \tan\theta_2 = -\cot\theta_1$

$\Rightarrow \quad \tan\theta_2 = \tan(90° + \theta_1)$

$\Rightarrow \quad \theta_2 = 90° + \theta_1$

$\Rightarrow \quad L_1 \perp L_2.$

Hence, two non-vertical lines are perpendicular to each other if and only if the product of their slopes is −1.

The Slope of a Line Passing Through the Points (x_1, y_1) and (x_2, y_2)

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points.

Let AB be the straight line passing through the points A and B.

Let θ be the inclination of the line $\overleftrightarrow{AB}$

Draw the perpendiculars AL and BM on to X-axis from A and B respectively. Also draw $AN \perp BM$.

Then, $\angle NAB = \theta$

Also, $BN = BM - MN = BM - AL = y_2 - y_1$

$AN = LM = OM - OL = x_2 - x_1$.

$\therefore$ The slope of the line L is, $m = \tan\theta = \dfrac{BN}{AN} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Hence, the slope of a line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

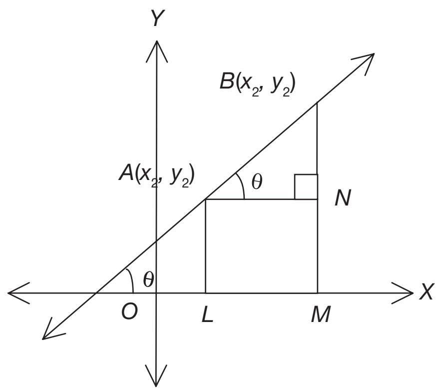
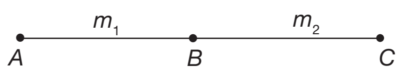


Figure 15.13

The following table gives the inclination (θ) of the line and its corresponding slope (m) for some particular values of θ.

θ	0°	30°	45°	60°	90°	120°	135°	150°
$m = \tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$\frac{-1}{\sqrt{3}}$

Note If the points A, B and C are collinear, then the slope of AB = the slope of BC.



That is, if $m_1 = m_2$, then A, B and C are collinear.

EXAMPLE 15.15

Find the slope of line joining the points (5, −3) and (7, −4).

SOLUTION

Let $A(5, -3)$ and $B(7, -4)$ be the given points.

Then, the slope of $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-4 - (-3)}{7 - 5} = \frac{-1}{2}.$$

EXAMPLE 15.16

Find the value of k if the slope of the line joining the points $(k, 4)$ and $(-3, -2)$ is $\frac{1}{2}$.

SOLUTION

Let the given points be $A(k, 4)$ and $B(-3, -2)$.

Given, the slope of $\overleftrightarrow{AB} = \frac{1}{2}$

$$\frac{-2-4}{-3-k} = \frac{1}{2}$$
$$\Rightarrow \quad -12 = -3 - k$$
$$\Rightarrow \quad k = -3 + 12$$
$$\Rightarrow \quad k = 9.$$

EXAMPLE 15.17

Find the value of m, if the line passing through the points $A(2, -3)$ and $B(3, m + 5)$ is perpendicular to the line passing through the points $P(-2, 3)$ and $Q(-4, -5)$.

SOLUTION

The slope of $\overleftrightarrow{AB}$, i.e., $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$.

$m_1 = m + 8$.

The slope of PQ, i.e., $(m_2) = \frac{y_2 - y_1}{x_2 - x_1} = 4$.

Since, $\overleftrightarrow{AB}$ and $\overleftrightarrow{PQ}$ are perpendicular to each other $\Rightarrow$ $m_1 m_2 = -1$, i.e.,

$$(m + 8) \times (4) = -1$$

$\Rightarrow \quad m + 8 = \frac{-1}{4}$.

Hence, $m = \frac{-33}{4}$.

EXAMPLE 15.18

If the points $(-3, 6)$, $(-9, a)$ and $(0, 15)$ are collinear, then find a.

SOLUTION

Let the given points be $A(-3, 6)$, $B(-9, a)$ and $C(0, 15)$.

The slope of $AB = \frac{a-6}{-9+3} = \frac{6-a}{6}$.

The slope of $BC = \frac{a-15}{-9-0} = \frac{15-a}{9}$.

Since the points A, B and C are collinear.

The slope of $\overleftrightarrow{AB} =$ the slope of $\overleftrightarrow{BC}$

$$\Rightarrow \quad \frac{6-a}{6} = \frac{15-a}{9}$$

$\Rightarrow \quad 18 - 3a = 30 - 2a$

$a = -12$.

Hence, $a = -12$.

Intercepts of a Straight Line

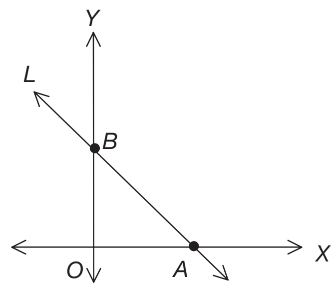


Figure 15.14

Say a straight line L meets X-axis in A and Y-axis in B.
Then, OA is called the x-intercept and OB is called the y-intercept.

Note OA and OB are taken as positive or negative based on whether the line meets positive or negative axes.

Example: The line L in the given figure meets X-axis at $A(4, 0)$ and Y-axis at $B(0, -5)$.

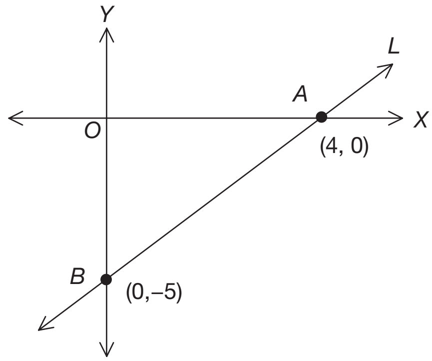


Figure 15.15

Hence, the x-intercept = 4 and y-intercept = –5.

Equation of a Line in General Form

An equation of the form, $ax + by + c = 0$. (where $a + b \neq 0$, i.e., a and b are not simultaneously equal to zero), which is satisfied by every point on a line and not by any point outside the line, is called the equation of a line.

Equations of Some Standard Lines

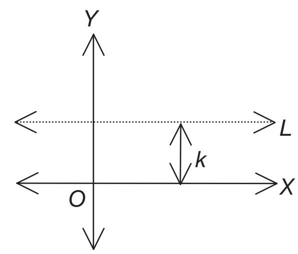


Figure 15.16

Equation of X-axis We know that the y-coordinate of every point on X-axis is zero so, if $P(x, y)$ is any point on X-axis, then $y = 0$.

Hence, the equation of X-axis is $y = 0$.

Equation of Y-axis We know that the x-coordinate of every point on Y-axis is zero. So, if $P(x, y)$ is any point on Y-axis, then $x = 0$, hence, the equation of Y-axis is $x = 0$.

Equation of a Line Parallel to X-axis Let L be a line parallel to X-axis and at a distance of k units away from X-axis.

Then, the y-coordinate of every point on the line L is k.

So, if $P(x, y)$ is any point on the line L, then $y = k$.

Hence, the equation of a line parallel to X-axis at a distance of k units from it is $y = k$.

Note For the lines lying below X-axis, k is taken as negative.

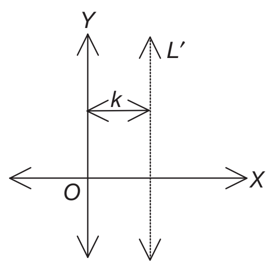


Figure 15.17

Equation of a Line Parallel to Y-axis Let L' be a line parallel to Y-axis and at a distance of k units away from it.

Then the x-coordinate of every point on the line L' is k.

So, if $P(x, y)$ is any point on the line L', then $x = k$.

Hence, the equation of a line parallel to Y-axis and at a distance of k units from it is $x = k$.

Note For the lines lying towards the left side of Y-axis, k is taken as negative.

Oblique Line

A straight line which is neither parallel to X-axis nor parallel to Y-axis is called an oblique line or an inclined line.

Different Forms of Equations of Oblique Lines

Gradiant Form (or) Slope Form The equation of a straight line with slope m and passing through origin is given by $y = mx$.

Point-Slope Form The equation of a straight line passing through the point (x_1, y_1) and with slope m is given by $y - y_1 = m(x - x_1)$.

Slope-intercept Form The equation of a straight line with slope m and having y-intercept c is given by $y = mx + c$.

Two-point Form The equation of a straight line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{or} \quad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}.$$

Intercept Form The equation of a straight line with x-intercept as a and y-intercept as b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

Note Area of triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}|ab|$ sq. units.

EXAMPLE 15.19

Find the equation of a line parallel to X-axis and passing through the point $(3, -4)$.

SOLUTION

We know that the equation of a line parallel to X-axis can be taken as $y = k$.

Given, the line passes through the point $(3, -4)$

$$\Rightarrow \quad k = -4.$$

Hence, the equation of the required line is $y = -4$, i.e., $y + 4 = 0$.

EXAMPLE 15.20

Find the equation of a line having a slope of $-\frac{3}{4}$ and passing through the point $(3, -4)$.

SOLUTION

We know that, the equation of a line passing through the point (x_1, y_1) and having a slope m is given by $y - y_1 = m(x - x_1)$.

Hence, the equation of the required line is

$$y - (-4) = -\frac{3}{4}(x - 3)$$

$$\Rightarrow \quad 4(y + 4) = -3(x - 3)$$

$$\Rightarrow \quad 3x + 4y + 7 = 0.$$

EXAMPLE 15.21

Find the equation of a line making intercepts 3 and −4 on the coordinate axes respectively.

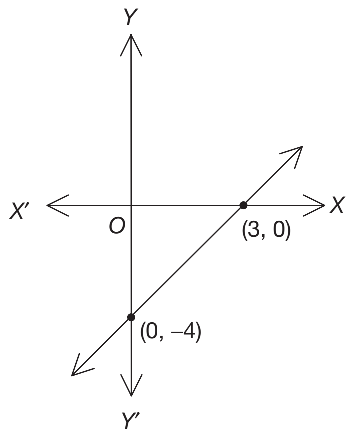


Figure 15.18

SOLUTION

Given,

x-intercept $(a) = 3$.

y-intercept $(b) = -4$.

∴ The equation of the required line is

$$\frac{x}{a} + \frac{y}{b} = 1,$$

$$\text{i.e., } \frac{x}{3} + \frac{y}{-4} = 1$$

$$4x - 3y = 12 \quad \text{(or)} \quad 4x - 3y - 12 = 0.$$

Area of Triangle

Consider $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ as the vertices of ΔABC.

Draw perpendiculars AP, BQ and CR on to X-axis.

Area of ΔABC = Area of trapezium $APQB$ + Area of trapezium $APRC$ − Area of trapezium $BCRQ$.

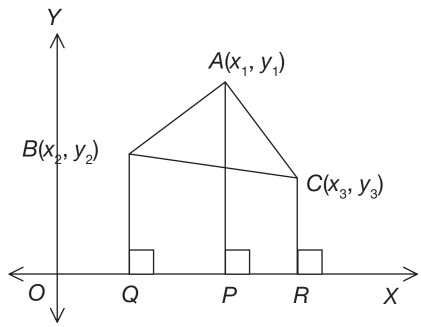


Figure 15.19

$$= \frac{1}{2}QP(AP + BQ) + \frac{1}{2}PR(AP + CR) - \frac{1}{2}QR(BQ + CR).$$

Here $QP = x_1 - x_2$, $PR = x_3 - x_1$, $QR = x_3 - x_2$,

$AP = y_1$, $BQ = y_2$, $CR = y_3$

$$= \frac{1}{2}(x_1 - x_2)(y_1 + y_2) + \frac{1}{2}(x_3 - x_1)(y_1 + y_3) - \frac{1}{2}(x_3 - x_2)(y_2 + y_3)$$

$$= \frac{1}{2}(x_1y_2 - x_1y_3 - x_2y_1 + x_2y_3 + x_3y_1 - x_3y_2)$$

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)].$$

As the area is always positive

∴ Area of ΔABC 'Δ' $= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$ sq. units.

'or'

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix} \text{ sq. units.}$$

Notes

1. Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and $(0, 0)$ is $\Delta = \frac{1}{2}|x_1y_2 - x_2y_1|$.
2. Area of ΔABC is zero, if the points A, B and C are collinear.
3. Area of triangle DEF formed by the mid-points of the sides of the ΔABC is $\frac{1}{4}$ of the area of ΔABC, i.e., Area of $\Delta ABC = 4$(Area of ΔDEF).
4. If G is the centroid of ΔABC, then Area of ΔABC = 3(Area of ΔAGB) = 3(Area of ΔBGC) = 3(Area of ΔACG).

Area of a Quadrilateral

Area of a quadrilateral with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) is given by $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$.

EXAMPLE 15.22

Find the area of the triangle whose vertices are $A(1, -2)$, $B(3, 4)$ and $C(2, 3)$.

SOLUTION

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1-3 & -2-4 \\ 3-2 & 4-3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -6 \\ 1 & 1 \end{vmatrix} = \frac{1}{2}|-2-(-6)| = 2$ sq. units.

EXAMPLE 15.23

Find the value of p, if the points $A(2, 3)$, $B(-1, 6)$ and $C(p, 4)$ are collinear.

SOLUTION

Given, the points $A(2, 3)$, $B(-1, 6)$ and $C(p, 4)$ are collinear.

$\therefore$ Area of $\Delta ABC = 0$.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2-(-1) & 3-6 \\ -1-p & 6-4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 & -3 \\ -1-p & 2 \end{vmatrix} = 0$$

$\Rightarrow 6 - 3(1 + p) = 0 \quad \Rightarrow \quad p = 1.$

Hence, $p = 1$.

Section Formulae

Section Formula

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$. Let $C(x, y)$ be any point on AB that divide AB in the ratio $m : n$.

Draw perpendiculars AL, CN and BM to X-axis.

AP and CQ are perpendiculars drawn to CN and BM.

Now it is clear that ΔAPC and ΔCQB are similar.

$$\therefore \quad \frac{AC}{CB} = \frac{AP}{CQ} = \frac{CP}{BQ} \qquad (1)$$

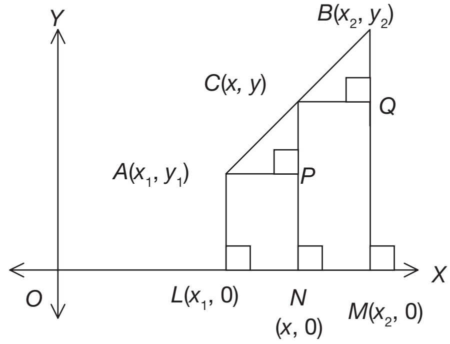


Figure 15.20

Here, $LN = x - x_1$ and $NM = x_2 - x$.

From Eq. (1), we have

$$\frac{m}{n} = \frac{x - x_1}{x_2 - x}, \quad (\text{AP} = \text{LN and CQ} = \text{NM})$$

$$\Rightarrow \quad mx_2 - mx = nx - nx_1.$$

$$\Rightarrow \quad mx_2 + nx_1 = mx + nx.$$

$$\therefore \quad x = \frac{mx_2 + nx_1}{m + n}.$$

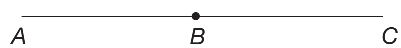
Similarly we can obtain $\frac{m}{n} = \frac{y - y_1}{y_2 - y} \quad \Rightarrow \quad y = \frac{my_2 + ny_1}{m + n}$.

Hence the coordinates of 'C' are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$.

In this case we notice that point C lies in between A and B. So we say that C divides AB in the ratio $m : n$ internally.

Notes

1. When C does not lie between A and B, i.e., as shown below, then we say that C divides AB in $m : n$ ratio externally, then the coordinates of C are $\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$.



2. Let $P(x, y)$ divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then, $\frac{m}{n} = \frac{x - x_1}{x_2 - x}$ (or) $\frac{y - y_1}{y_2 - y}$.
3. X-axis divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $-y_1 : y_2$ (or) $y_1 : -y_2$.
5. Y-axis divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $-x_1 : x_2$.

EXAMPLE 15.24

Find the coordinates of the point P which divides the line segment joining the points $A(3, -2)$ and $B(2, 6)$ internally in the ratio 2 : 3.

SOLUTION

Given, $P(x, y)$ divides AB internally in the ratio 2 : 3.

$$\text{So,} \quad P = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right).$$

Here, $(x_1, y_1) = (3, -2)$, $(x_2, y_2) = (2, 6)$ and $m : n = 2 : 3$

$$\therefore \quad P = \left(\frac{2(2) + 3(3)}{2 + 3}, \frac{2(6) + 3(-2)}{2 + 3}\right) = \left(\frac{13}{5}, \frac{6}{5}\right).$$

Hence, $P\left(\frac{13}{5}, \frac{6}{5}\right)$ is the required point.

EXAMPLE 15.25

Find the coordinates of a point P which divides the line segment joining the points $A(1, 3)$ and $B(3, 4)$ externally in the ratio 3 : 4.

SOLUTION

Given, P divides AB externally in the ratio 3 : 4.

So, $P = \left(\dfrac{mx_2 - nx_1}{m - n}, \dfrac{my_2 - ny_1}{m - n}\right)$.

Here, $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (3, 4)$ and $m : n = 3 : 4$.

$\therefore \quad P = \left(\dfrac{3(3) - 4(1)}{3 - 4}, \dfrac{3(4) - 4(3)}{3 - 4}\right) = (-5, 0)$.

Hence, $P = (-5, 0)$.

EXAMPLE 15.26

Find the ratio in which the point $P(3, -2)$ divides the line segment joining the points $A(1, 2)$ and $B(-1, 6)$.

SOLUTION

The ratio in which P divides AB is $AP : PB = (3 - 1) : (-1 - 3) = 2 : -4 = -1 : 2$.

Hence, P divides AB externally in the ratio 1 : 2.

EXAMPLE 15.27

Find the ratio in which the line joining the points $(2, -3)$ and $(3, 1)$ is divided by X-axis and Y-axis.

SOLUTION

The ratio in which X-axis divides is $-y_1 : y_2$,

i.e., $-(-3) : 1 = 3 : 1$

The ratio in which Y-axis divides is $-x_1 : x_2$

$= -2 : 3$

That is, 2 : 3 externally.

Mid-point

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points and M be the mid-point of AB.

Then, M divides AB in the ratio 1 : 1 internally.

So, $M = \left(\dfrac{1 \cdot x_2 + 1 \cdot x_1}{1 + 1}, \dfrac{1 \cdot y_2 + 1 \cdot y_1}{1 + 1}\right) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Hence, the coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are given by $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Points of Trisection

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points. Then, the two points which divide AB in the ratio 1 : 2 and 2 : 1 internally are called the points of trisection of AB.

Further, if P and Q are the points of trisection of AB respectively,

$$\text{Then } P = \left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3}\right) \text{ and } Q = \left(\frac{x_1 + 2x_2}{3}, \frac{y_1 + 2y_2}{3}\right).$$

Notes

1. If the mid-points of ΔABC are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$, then its vertices are $A(-x_1 + x_2 + x_3, -y_1 + y_2 + y_3)$, $B(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$ and $C(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$.
2. The fourth vertex of a parallelogram whose three vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) in order is $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.

EXAMPLE 15.28

Find the mid-point of the line segment joining the points (1, −3) and (6, 5).

SOLUTION

Let $A(1, -3)$ and $B(6, 5)$ be the given points and M be the mid-point of AB.

$$\text{Then, } M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+6}{2}, \frac{-3+5}{2}\right) = \left(\frac{7}{2}, 1\right).$$

Hence, the mid-point of AB is $\left(\frac{7}{2}, 1\right)$.

EXAMPLE 15.29

Find the points of trisection of the line segment joining the points (3, −2) and (4, 1).

SOLUTION

Let $A(3, -2)$ and $B(4, 1)$ be the given points.

Let P and Q be the points of trisection of AB respectively.

$$\text{Then, } P = \left(\frac{2(3)+4}{3}, \frac{2(-2)+1}{3}\right) \text{ and } Q = \left(\frac{3+2(4)}{3}, \frac{-2+2(1)}{3}\right)$$

$$\Rightarrow \quad P = \left(\frac{10}{3}, -1\right) \text{ and } Q = \left(\frac{11}{3}, 0\right).$$

Hence, the points of trisection are $\left(\frac{10}{3}, -1\right)$ and $\left(\frac{11}{3}, 0\right)$.

EXAMPLE 15.30

Find the fourth vertex of the rhombus formed by (−1, −1), (6, 1) and (8, 8).

SOLUTION

Let the three vertices of rhombus be $A(-1, -1)$, $B(6, 1)$ and $C(8, 8)$, then fourth vertex $D(x, y)$ is given by,

$$D(x, y) = (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$$
$$= (-1 - 6 + 8, -1 - 1 + 8)$$
$$= (1, 6).$$

Hence, the required fourth vertex is $D(1, 6)$.

EXAMPLE 15.31

Find the area of the triangle formed by the mid-points of the sides of ΔABC, where $A(3, 2)$, $B(-5, 6)$ and $C = (8, 3)$.

SOLUTION

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 3-(-5) & 2-6 \\ -5-8 & 6-3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & -4 \\ -13 & 3 \end{vmatrix} = \frac{1}{2}|8(3) - 4(13)| = \frac{1}{2}|24 - 52|$

$= \frac{1}{2}|-28| = 14$ sq. units.

Hence the area of triangle formed by the mid-points of the sides of $\Delta ABC = \frac{1}{4}$(Area of ΔABC)

$= \frac{1}{4}(14) = 3.5$ sq. units.

Centroid

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC and G be its centroid. Then, the coordinates of G are given by, $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

EXAMPLE 15.32

Find the centroid of ΔABC whose vertices, are $A(1, -3)$, $B(-3, 6)$ and $C(-4, 3)$.

SOLUTION

Given, $A(1, -3)$, $B(-3, 6)$ and $C(-4, 3)$.

So, centroid of $\Delta ABC = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{1-3-4}{3}, \frac{-3+6+3}{3}\right) = (-2, 2)$.

Hence, $(-2, 2)$ is the centroid of ΔABC.

EXAMPLE 15.33

Find the centroid of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 10$ as sides.

SOLUTION

Let OAB be the triangle formed by the given lines.

O is the point of intersections of $x = 0$ and $y = 0$, i.e., origin $\Rightarrow$ $O(0, 0)$.

A is the point of intersection of $x = 0$ and $x + y = 10$, i.e., $A(0, 10)$ and B is the point of intersection of $y = 0$ and $x + y = 10$, i.e., $B(10, 0)$.

$\therefore$ Centroid of ΔOAB is

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
$$= \left(\frac{0+0+10}{3}, \frac{0+10+0}{3}\right)$$
$$= \left(\frac{10}{3}, \frac{10}{3}\right).$$

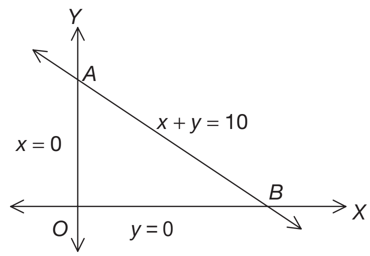


Figure 15.21

EXAMPLE 15.34

Find the third vertex of ΔABC if two of its vertices are $A(-3, 2)$, $B(1, 5)$ and its centroid is $G(3, -4)$.

SOLUTION

Let $C(x, y)$ be the third vertex.

Given, centroid of $\Delta ABC = (3, -4)$

$$\Rightarrow \quad \left(\frac{-3+1+x}{3}, \frac{2+5+y}{3}\right) = (3, -4)$$

$$\Rightarrow \quad \frac{-2+x}{3} = 3, \frac{7+y}{3} = -4$$

$\Rightarrow \quad x = 11, y = -19.$

$\therefore$ The third vertex is $(11, -19)$.

Equation of a Line Parallel or Perpendicular to the Given Line

Let $ax + by + c = 0$ be the equation of a straight line, then,

1. The equation of a line passing through the point (x_1, y_1) and parallel to the given line is given by $a(x - x_1) + b(y - y_1) = 0$.
2. The equation of a line passing through the point (x_1, y_1) and perpendicular to the given line is given by $b(x - x_1) - a(y - y_1) = 0$.

EXAMPLE 15.35

Find the equation of a line passing through the point $A(2, -3)$ and parallel to the line $2x - 3y + 6 = 0$.

SOLUTION

Here, $(x_1, y_1) = (2, -3)$, $a = 2$ and $b = -3$.

$\therefore$ Equation of the line passing through $A(2, -3)$ and parallel to the line $2x - 3y + 6 = 0$ is $a(x - x_1) + b(y - y_1) = 0$, i.e.,

$$2(x - 2) - 3(y + 3) = 0$$
$$\Rightarrow \quad 2x - 3y - 13 = 0.$$

Hence, the equation of the required line is $2x - 3y - 13 = 0$.

EXAMPLE 15.36

Find the equation of a line passing through the point (5, 2) and perpendicular to the line $3x - y + 6 = 0$.

SOLUTION

Here, $(x_1, y_1) = (5, 2)$, $a = 3$ and $b = -1$.

$\therefore$ Equation of the line perpendicular to $3x - y + 6 = 0$ and passing through the point (5, 2) is $b(x - x_1) - a(y - y_1) = 0$, i.e.,

$$-1(x - 5) - 3(y - 2) = 0$$
$$\Rightarrow \quad (x - 5) + 3(y - 2) = 0$$
$$\Rightarrow \quad x + 3y - 11 = 0.$$

Hence, the equation of the required line is $x + 3y - 11 = 0$.

EXAMPLE 15.37

The line $(5x - 8y + 11)\lambda + (8x - 3y + 4) = 0$ is parallel to X-axis. Find λ.

SOLUTION

The given line is

$(5x - 8y + 11)\lambda + (8x - 3y + 4) = 0$

That is, $x(5\lambda + 8) - y(8\lambda + 3) + (11\lambda + 4) = 0$.

As the given line is parallel to X-axis, its slope = 0, i.e.,

$$\frac{-(5\lambda + 8)}{-(8\lambda + 3)} = 0$$
$$\Rightarrow \quad 5\lambda + 8 = 0.$$

Hence, $\lambda = \dfrac{-8}{5}$.

Median of the Triangle

A line drawn from the vertex, which bisects the opposite side is called a median of the triangle.

EXAMPLE 15.38

Find the median to the side BC of the triangle whose vertices are $A(-2, 1)$, $B(2, 3)$ and $C(4, 5)$.

SOLUTION

Let E be the mid-point of side BC.

$$\therefore \quad E = \left(\frac{2+4}{2}, \frac{3+5}{2}\right) = (3, 4)$$

Equation of line AE is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 1 = \frac{4-1}{3-(-2)}(x+2).$$

$$y - 1 = \frac{3}{5}(x+2)$$

$$5y - 5 = 3x + 6.$$

$\therefore$ The required equation of the median is

$3x - 5y + 11 = 0.$

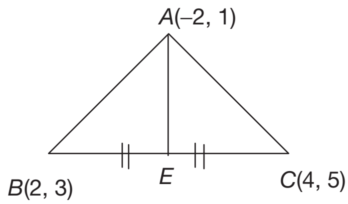


Figure 15.22

Altitude of the Triangle

A perpendicular dropped from the vertex to the opposite side in a triangle is called an altitude.

EXAMPLE 15.39

Find the equation of the altitude drawn to side BC of ΔABC, whose vertices are $A(-2, 1)$, $B(2, 3)$ and $C(4, 5)$.

SOLUTION

Let AD be the altitude drawn to side BC.

Slope of $BC = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-3}{4-2} = 1.$

$\therefore$ Slope of $AD = -1$ ($\because$ $AD \perp BC$)

Equation of AD is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -1(x + 2)$$
$$y - 1 = -x - 2$$

$\Rightarrow \quad x + y + 1 = 0.$

$\therefore$ The required equation of altitude is $x + y + 1 = 0$.

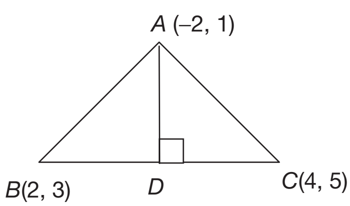


Figure 15.23

EXAMPLE 15.40

Find the equation of the perpendicular bisector drawn to the side BC of ΔABC, whose vertices are $A(-2, 1)$, $B(2, 3)$ and $C(4, 5)$.

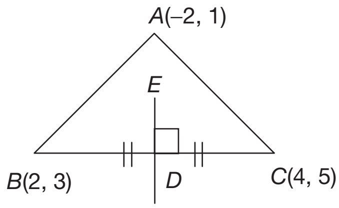


Figure 15.24

SOLUTION

$\therefore \quad D = \left(\dfrac{2+4}{2}, \dfrac{3+5}{2}\right)$

$D = (3, 4)$

Slope of $BC = \dfrac{5-3}{4-2} = 1$

$\Rightarrow$ Slope of $ED = -1$ ($\because ED \perp BC$)

Equation of ED is $y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - 4 = -1(x - 3)$$

$$\Rightarrow \quad y - 4 = -x + 3$$

$$\Rightarrow \quad x + y - 7 = 0.$$

$\therefore$ The required equation of the perpendicular bisector is $x + y - 7 = 0$.

TEST YOUR CONCEPT

Very Short Answer Type Questions

1. If the inclination of a line is 45°, then the slope of the line is _______.
2. If the point (x, y) lies in the third quadrant, then x is _______ and y is _______.
3. The point of intersection of the lines $x = 2$ and $y = 3$ is _______.
4. The point of intersection of medians of triangle is called its _______.
5. The line $y + 7 = 0$ is parallel to _______ axis.
6. The distance of a point (2, 3) from Y-axis is _______.
7. The slope of the line perpendicular to $5x + 3y + 1 = 0$ is _______.
8. If (x, y) represents a point and $|x| > 0$ and $y > 0$ then in which quadrants can the point lie?
9. The line $4x + 7y + 9 = 0$ meet the X-axis at _______ and Y-axis at _______.
10. If (x, y) represents a point and $xy < 0$, then the point may lie in _______ or _______ quadrant.
11. The area of the triangle, with vertices $A(2, 0)$, $B(0, -4)$ and origin is _______.
12. The area of the triangle formed by the points (0, 0), (0, a), (b, 0) is _______.
13. The area of the triangle formed by the line $ax + by + c = 0$ with coordinate axes is _______.
14. If the points (5, 5), (7, 7) and (a, 8) are collinear, then the value of a is _______.
15. Area of a triangle whose vertices are (0, 0), (x_1, y_1), (x_2, y_2) is _______.
16. The end vertices of one diagonal of a parallelogram are (1, 3) and (5, 7), then the mid-point of the other diagonal is _______.
17. The ortho-centre of the triangle formed by the points (0, 1), (1, 2) and (0, 2) is _______.
18. The two straight lines, $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular to each other, then $m_1m_2 =$ _______.
19. If $A(4, 0)$, $B(0, -6)$ are the two vertices of a triangle OAB where O is origin, then the circumcentre of the triangle OAB is _______.
20. If the centre and radius of a circle is (3, 4) and 7, then the position of the point (5, 3) wrt the circle is _______.
21. If $A(2, 3)$, $B(x, y)$ and $C(4, 3)$ are the vertices of a right triangle, right angled at A, then $x =$ _______.
22. The coordinates of the points P which divides (1, 0) and (0, 0) in 1 : 2 ratio are _______.
23. The points A, B and C represents the vertices of a ΔABC. AD is the median drawn from A to BC, then the centroid of the triangle divides AD in _______ ratio.
24. If (5, 7) and (9, 3) are the ends of the diameter of a circle, then the centre of the circle is _______.
25. What is the formula for calculating the area of a quadrilateral whose vertices are (x_1, y_1),(x_2, y_2), (x_3, y_3), (x_4, y_4)?
26. The centroid of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 6$ is _______.
27. The line $ax + by + c = 0$ is such that $a = 0$ and $bc \neq 0$, then the line is perpendicular to _______ axis.
28. If A is one of the points of trisection of the line joining B and C, then A divides BC in the ratio _______ (or) _______.
29. The ratio that the line joining points (3, −6) and (4, 9) is divided by X-axis is _______.
30. If (1, 2), (3, 4) and (0, 6) are the three vertices of a parallelogram taken in that order, then the fourth vertex is _______.

Short Answer Type Questions

31. Find λ, if the line $(3x - 2y + 5) + \lambda(3x - y + 4) = 0$ passes through the mid-point of the line joining the points $A(2, 3)$ and $B(4, 9)$.
32. Find the distance between the points (3, −5) and (−4, 7).

33. Find the slope of the line perpendicular to $\overleftrightarrow{AB}$ where $A(5, -6)$ and $B(2, -7)$.

34. If $A(-2, -1)$, $B(-4, 5)$ and $C(2, 3)$ are three vertices of the parallelogram $ABCD$, then find the vertex D.

35. Find the equation of line parallel to Y-axis and passing through the point $(3, -5)$.

36. Find the equation of a line, whose inclination is $30°$ and making an intercept of $\frac{-3}{5}$ on Y-axis.

37. Find the equation of a straight line whose slope is -5 and making an intercept 3 on the Y-axis.

38. Find the centroid of a triangle whose vertices are $(3, -1)$, $(2, 4)$ and $(-8, 6)$.

39. Find the equation of a line passing through the point $(2, -3)$ and parallel to the line $2x - 3y + 8 = 0$.

40. If $A(2, -2)$, $B(3, 4)$ and $C(7, 2)$ are the mid points of the sides PQ, QR and RP respectively of ΔPQR, then find its vertices.

41. Let $(-3, 2)$ be one end of a diameter of a circle with centre $(4, 6)$. Find the other end of the diameter.

42. Let $A(-1, 2)$ and $D(3, 4)$ be the ends points of the median AD of ΔABC. Find the centroid of ΔABC.

43. Find the point of intersection of the lines $3x + 5y + 2 = 0$ and $4x + 7y + 3 = 0$.

44. Find the coordinates of the point which divides the line joining the points $A(-3, 2)$ and $B(2, 6)$ internally in the ratio $3 : 2$.

45. Find the point on Y-axis which is equidistant from $A(3, -6)$ and $B(-2, 5)$.

Essay Type Questions

46. Find the equation of a line parallel to the line $2x + 3y - 6 = 0$ and where sum of intercepts is 10.

47. Find the equation of a line passing through the point of intersection of the lines $5x - y - 7 = 0$ and $3x - 2y - 7 = 0$ and parallel to the $x + 3y - 5 = 0$.

48. Find the equation of a line with y-intercept -4 and perpendicular to a line passing through the points $A(1, -2)$ and $B(-3, 2)$.

49. Find the circum-centre of the triangle formed by the points $(2, 3)$, $(1, -5)$ and $(-1, 4)$.

50. Let $A(3, 2)$, $B(-4, 1)$, $C(-3, 1)$ and $D(2, -4)$ be the vertices of a quadrilateral $ABCD$. Find the area of the quadrilateral formed by the mid-points of the sides of the quadrilateral $ABCD$.

CONCEPT APPLICATION

Level 1

1. The lines, $x = 2$ and $y = 3$ are
   (a) parallel to each other.
   (b) perpendicular to each other.
   (c) neither parallel nor perpendicular to each other.
   (d) None of these

2. The lines, $x = -2$ and $y = 3$ intersect at the point ______.
   (a) $(-2, 3)$ (b) $(2, -3)$
   (c) $(3, -2)$ (d) $(-3, 2)$

3. The slope of the line joining the points $(2, k - 3)$ and $(4, -7)$ is 3. Find k.
   (a) -10 (b) -6
   (c) -2 (d) 10

4. The centre of a circle is $C(2, -3)$ and one end of the diameter AB is $A(3, 5)$. Find the coordinates of the other end B.
   (a) $(1, -11)$ (b) $(5, 2)$
   (c) $(1, 8)$ (d) None of these

5. The angle made by the line $\sqrt{3}x - y + 3 = 0$ with the positive direction of X-axis is ______.

(a) $30°$ (b) $45°$

(c) $60°$ (d) $90°$

6. The points on X-axis which are at a distance of $\sqrt{13}$ units from $(-2, 3)$ is _______.

(a) $(0, 0), (-2, -3)$ (b) $(0, 0), (-4, 0)$

(c) $(0, 0), (2, 3)$ (d) None of these

7. The point P lying in the fourth quadrant which is at a distance of 4 units from X-axis and 3 units from Y-axis is _______.

(a) $(4, -3)$ (b) $(4, 3)$

(c) $(3, -4)$ (d) $(-3, 4)$

8. The radius of a circle with centre $(-2, 3)$ is 5 units. The point $(2, 5)$ lies

(a) on the circle.

(b) inside the circle.

(c) outside the circle.

(d) None of these

9. The points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$

(a) are collinear.

(b) form a scalene triangle.

(c) form an equilateral triangle.

(d) None of these

10. Find λ, if the line $3x - \lambda y + 6 = 0$ passes through the point $(-3, 4)$.

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

11. If $A(-2, 3)$ and $B(2, 3)$ are two vertices of ΔABC and $G(0, 0)$ is its centroid, then the coordinates of C are

(a) $(0, -6)$ (b) $(-4, 0)$

(c) $(4, 0)$ (d) $(0, 6)$

12. Let ΔABC be a right triangle in which $A(0, 2)$ and $B(2, 0)$. Then the coordinates of C can be

(a) $(0, 0)$

(b) $(2, 2)$

(c) Either (a) or (b)

(d) None of these

13. If $A(4, 7)$, $B(2, 5)$, $C(1, 3)$ and $D(-1, 1)$ are the four points, then the lines AC and BD are _______.

(a) perpendicular to each other

(b) parallel to each other

(c) neither parallel nor perpendicular to each other

(d) None of these

14. Find the area of the triangle formed by the line $5x - 3y + 15 = 0$ with coordinate axes.

(a) 15 cm^2 (b) 5 cm^2

(c) 8 cm^2 (d) $\frac{15}{2} \text{ cm}^2$

15. Equation of a line whose inclination is $45°$ and making an intercept of 3 units on X-axis is

(a) $x + y - 3 = 0$

(b) $x - y - 3 = 0$

(c) $x - y + 3 = 0$

(d) $x + y + 3 = 0$

16. The centre of a circle is $C(2, k)$. If $A(2, 1)$ and $B(5, 2)$ are two points on its circumference, then the value of k is

(a) 6 (b) 2

(c) -6 (d) -2

17. The lines $x = -1$ and $y = 4$ are _______.

(a) perpendicular to each other

(b) parallel to each other

(c) neither parallel nor perpendicular to each other

(d) None of these

18. The distance between the points $(2k + 4, 5k)$ and $(2k, -3 + 5k)$ in units is

(a) 1 (b) 2

(c) 4 (d) 5

19. The equation of the line with inclination $45°$ and passing through the point $(-1, 2)$ is

(a) $x + y + 3 = 0$ (b) $x - y + 3 = 0$

(c) $x - y - 3 = 0$ (d) $x + y - 3 = 0$

20. The distance between the points $(3k + 1, -3k)$ and $(3k - 2, -4 - 3k)$ (in units) is

(a) $3k$ (b) $5k$

(c) 5 (d) 3

PRACTICE QUESTIONS

21. The angle made by the line $x - \sqrt{3}y + 1 = 0$ with the positive Y-axis is

(a) 60° (b) 30°

(c) 45° (d) 90°

22. If ΔABC is a right triangle in which $A(3, 0)$ and $B(0, 5)$, then the coordinates of C can be

(a) (5, 3) (b) (3, 5)

(c) (0, 0) (d) Both (b) and (c)

23. If the roots of the quadratic equation $2x^2 - 5x + 2 = 0$ are the intercepts made by a line on the coordinate axes, then the equation of the line can be

(a) $4x + y = 2$ (b) $2x + 5y + 2$

(c) $x + 4y = 2$ (d) Both (a) and (c)

24. The inclination of the line $\sqrt{3}x - y + 5 = 0$ with X-axis is

(a) 90° (b) 45°

(c) 60° (d) 30°

25. The equation of the line parallel to $3x - 2y + 7 = 0$ and making an intercept -4 on X-axis is

(a) $3x - 2y + 12 = 0$

(b) $3x - 2y - 12 = 0$

(c) $3x + 2y - 12 = 0$

(d) $3x + 2y + 12 = 0$

26. A triangle is formed by the lines $x + y = 8$, X-axis and Y-axis. Find its centroid.

(a) $\left(\frac{8}{3}, \frac{8}{3}\right)$ (b) (8, 8)

(c) (4, 4) (d) (0, 0)

27. The point which divides the line joining the points $A(1, 2)$ and $B(-1, 1)$ internally in the ratio 1 : 2 is _______.

(a) $\left(\frac{-1}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{5}{3}\right)$

(c) (−1, 5) (d) (1, 5)

28. Find the area of the triangle formed by the line $3x - 4y + 12 = 0$ with the coordinate axes.

(a) 6 units^2 (b) 12 units^2

(c) 1 units^2 (d) 36 units^2

29. The equation of a line whose sum of intercepts is 5 and the area of the triangle formed by the line with positive coordinate axis is 2 sq. units can be

(a) $x + y = 4$ (b) $x + 4y = 4$

(c) $y + 4x + 4 = 0$ (d) $y = x + 4$

30. Find the equation of a line which divides the line segment joining the points (1, 1) and (2, 3) in the ratio 2 : 3 perpendicularly.

(a) $5x - 5y + 2 = 0$

(b) $5x + 5y + 2 = 0$

(c) $x + 2y - 5 = 0$

(d) $x + 2y + 7 = 0$

Level 2

31. The equation of the line making an angle of 45° with X-axis in positive direction and having y-intercept as -3 is

(a) $3x - y + 1 = 0$

(b) $3x + y - 1 = 0$

(c) $x - y + 3 = 0$

(d) $x - y = 3$

32. The ratio in which the line joining points $(a + b, b + a)$ and $(a - b, b - a)$ is divided by the point(a, b) is _______.

(a) $b : a$ internally (b) 1 : 1 internally

(c) $a : b$ externally (d) 2 : 1 externally

33. Which of the following lines is perpendicular to $x + 2y + 3 = 0$?

(a) $2\sqrt{2}x - y + 3 = 0$

(b) $\sqrt{2}x + \sqrt{2}y - 5 = 0$

(c) $2\sqrt{2}x - \sqrt{2}y + 3 = 0$

(d) $x + \sqrt{2}y + 4 = 0$

34. The ortho-centre of the triangle formed by the points (0, 3), (0, 0) and (1, 0) is

(a) $\left(\frac{1}{3}, 1\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) (0, 0) (d) $\left(0, \frac{3}{2}\right)$

35. The equation of the line passing through the point of intersection of the lines $x + 2y + 3 = 0$ and $2x - y + 5 = 0$ and parallel to X-axis is

(a) $5y + 1 = 0$ (b) $5x - 13 = 0$

(c) $5x + 13 = 0$ (d) $5y - 1 = 0$

36. Find the equation of a line which divides the line segment joining the points (1, –2) and (3, –1) in the ratio 3 : 1 perpendicularly.

(a) $x - 2y - 5 = 0$

(b) $6x + 4y - 5 = 0$

(c) $3x + 2y - 5 = 0$

(d) $8x + 4y - 15 = 0$

37. If $x + p = 0$, $y + 4 = 0$ and $x + 2y + 4 = 0$ are concurrent, then $p =$

(a) 4 (b) –2

(c) –4 (d) 2

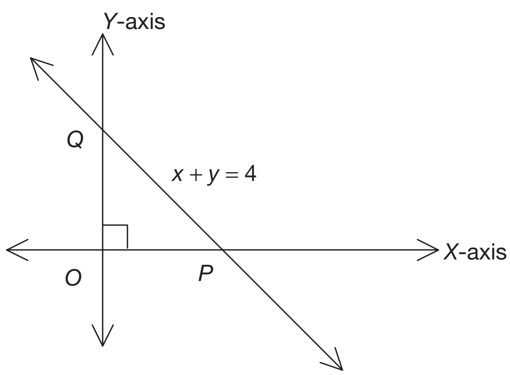
38. The perpendicular bisector of the side PQ is

(a) $x - y = 0$

(b) $x + y - 2 = 0$

(c) $3x - 2y - 2 = 0$

(d) $x + 2y - 6 = 0$



39. The ortho-centre of the triangle formed by the vertices $A(4, 6)$, $B(4, 3)$ and $C(2, 3)$ is

(a) (2, 3) (b) (4, 3)

(c) (4, 6) (d) (3, 4)

40. The circum-centre and the ortho-centre of the triangle formed by the sides $y = 0$, $x = 0$ and $2x + 3y = 6$ are respectively

(a) (3, 2), (0, 0) (b) $\left(\frac{3}{2}, 1\right)$, (0, 0)

(c) (–3, 1), (0, 0) (d) $\left(-\frac{3}{2}, 1\right)$, (0, 0)

41. The equation of median drawn to the side BC of ΔABC whose vertices are $A(1, -2)$, $B(3, 6)$ and $C(5, 0)$ is

(a) $5x - 3y - 11 = 0$

(b) $5x + 3y - 11 = 0$

(c) $3x - 5y + 11 = 0$

(d) $3x - 5y - 11 = 0$

42. Find the equation of the line passing through (1, 1) and forming an area of 2 sq. units with positive coordinate axis.

(a) $2x + 3y = 5$ (b) $x - y + 2 = 0$

(c) $x + y - 2 = 0$ (d) $x - y + 1 = 0$

43. If the vertices of a triangle are $A(3, -3)$, $B(-3, 3)$ and $C(-3\sqrt{3}, -3\sqrt{3})$, then the distance between the ortho-centre and the circum-centre is

(a) $6\sqrt{2}$ units (b) $6\sqrt{3}$ units

(c) 0 unit (d) None of these

44. The circum-centre of the triangle formed by the lines $x + 4y = 7$, $5x + 3y = 1$ and $3x - 5y = 21$ is

(a) (–3, 2) (b) (3, 1)

(c) (3, –1) (d) (–3, –2)

45. If the line $(3x - 8y + 5) + a(5x - 3y + 10) = 0$ is parallel to X-axis, then a is

(a) $-\frac{8}{3}$ (b) $-\frac{3}{5}$

(c) –2 (d) $-\frac{1}{2}$

46. Find the equation of the median of the triangle, formed by the line $8x + 5y = 40$ with the coordinate axes. Given that the median is passing through the origin.

(a) $5x - 8y = 0$ (b) $5x + 8y = 0$

(c) $8x - 5y = 0$ (d) $8x + 5y = 0$

PRACTICE QUESTIONS

47. Find the area of a triangle formed by the lines $4x - y - 8 = 0$, $2x + y - 10 = 0$ and $y = 0$ (in sq units).

(a) 5 (b) 6

(c) 4 (d) 3

48. Find the length of the longest side of the triangle formed by the line $3x + 4y = 12$ with the coordinate axes.

(a) 9 (b) 16

(c) 5 (d) 7

49. The following are the steps involved in finding the area of the triangle with vertices (2, 3), (4, 7) and (8, 5). Arrange them in the sequential order from first to last.

(A) $\frac{1}{2}|(-2)2-(-4)(-4)| = \frac{1}{2}|-20| = 10$ sq. units

(B) Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$

(C) Let $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (4, 7)$ and $(x_3, y_3) = (8, 5)$

(D) Area of triangle $= \frac{1}{2}\begin{vmatrix} 2-4 & 4-8 \\ 3-7 & 7-5 \end{vmatrix}$

(a) ABCD (b) CBDA

(c) CBAD (d) None of these

50. The following are the steps involved in finding the centroid of the triangle with vertices (5, 4), (6,7) and (1, 1). Arrange them in sequential order from first to last.

(A) The required centroid (4, 4).

(B) The centroid of the triangle with the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

(C) The centroid is $\left(\frac{5+6+1}{3}, \frac{4+7+1}{3}\right)$.

(a) ABC (b) CBA

(c) BAC (d) BCA

Level 3

51. In what ratio does the line $4x + 3y - 13 = 0$ divide the line segment joining the points (2, 1) and (1, 4)?

(a) 3 : 2 internally

(b) 2 : 3 externally

(c) 2 : 3 internally

(d) 3 : 2 externally

52. If $A(3, 4)$, $B(1, -2)$ are the two vertices of triangle ABC and $G(3, 5)$ is the centroid of the triangle, then the equation of AC is

(a) $4x - 5y - 7 = 0$

(b) $4x - 5y + 8 = 0$

(c) $9x - 2y - 23 = 0$

(d) $9x - 2y - 19 = 0$

53. If $ax + 4y + 3 = 0$, $bx + 5y + 3 = 0$ and $cx + 6y + 3 = 0$ are concurrent lines, then $a + c =$

(a) $3b$ (b) $2b$

(c) b (d) $4b$

54. If (5, 3), (4, 2) and (1, –2) are the mid points of sides of triangle ABC, then the area of ΔABC is

(a) 2 sq. units (b) 3 sq. units

(c) 1 sq. unit (d) 4 sq. units

55. Find the distance between the ortho-centre and circum-centre of the triangle formed by joining the points (5, 7), (4, 10) and (6, 9).

(a) $\sqrt{\frac{5}{4}}$ units (b) $\sqrt{\frac{5}{2}}$ units

(c) $\sqrt{10}$ units (d) $\sqrt{5}$ units

56. Area of the region formed by $4|x| + 3|y| = 12$ is ______.

(a) 18 sq. units (b) 20 sq. units

(c) 24 sq. units (d) 36 sq. units

57. Find the radius of the circle which passes through the origin, (0, 4) and (4, 0).

(a) 2 (b) $4\sqrt{2}$

(c) $\sqrt{8}$ (d) $3\sqrt{2}$

58. Find the circum-centre of the triangle whose vertices are $(0, 0)$, $(3, \sqrt{3})$ and $(0, 2\sqrt{3})$.

(a) $(1, \sqrt{3})$ (b) $(\sqrt{3}, \sqrt{3})$

(c) $(2\sqrt{3}, 1)$ (d) $(2, \sqrt{3})$

59. Find the ortho-centre of the triangle formed by the lines $3x - 4y = 10$, $8x + 6y = 15$ and Y-axis.

(a) $\left(\frac{3}{5}, \frac{7}{5}\right)$ (b) $\left(\frac{12}{5}, \frac{-7}{10}\right)$

(c) $\left(\frac{3}{5}, \frac{12}{5}\right)$ (d) $\left(\frac{12}{5}, \frac{-3}{10}\right)$

60. The equation of a line which passes through $(2, 3)$ and the product of whose intercepts on the coordinate axis is 27, can be

(a) $5x + 4y = 22$

(b) $3x - y = 3$

(c) $3x + 4y = 18$

(d) $2x + 3y = 13$

TEST YOUR CONCEPT

Very Short Answer Type Questions

1. 1
2. negative, negative.
3. (2, 3)
4. centroid
5. X-axis
6. 2 units.
7. $\frac{3}{5}$
8. first quadrant or second quadrant.
9. $\left(-\frac{9}{4}, 0\right)$ and $\left(0, -\frac{9}{7}\right)$
10. second quadrant or fourth quadrant
11. 4 sq. units
12. $\frac{1}{2}|ab|$ sq. units
13. $\frac{1}{2}\left|\frac{c^2}{ab}\right|$
14. 8
15. $\frac{1}{2}|x_1y_2 - x_2y_1|$
16. (3, 5)
17. (0, 2)
18. –1
19. (2, –3)
20. inside the circle
21. 2
22. $\left(\frac{2}{3}, 0\right)$
23. 2 : 1
24. (7, 5)
25. $\frac{1}{2}|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)|(y_1 - y_3)|$ sq. units
26. (2, 2)
27. Y-axis
28. 2 : 1; 1 : 2
29. 2 : 3
30. (–2, 4)

Short Answer Type Questions

31. $\left(\frac{-9}{10}, -\frac{7}{10}\right)$
32. $\sqrt{193}$ units
33. –3
34. (4, –3)
35. $x = 3$
36. $5\sqrt{3}y - 5x + 3\sqrt{3} = 0$.
37. $5x + y - 3 = 0$
38. (–1, 3)
39. $2x - 3y - 13 = 0$
40. $P(6, -4)$, $Q(-2, 0)$ and $R(8, 8)$.
41. (11, 10)
42. $\left(\frac{5}{3}, \frac{10}{3}\right)$
43. (1 – 1)
44. $\left(0, \frac{22}{5}\right)$
45. $\left(0, \frac{-8}{11}\right)$

Essay Type Questions

46. $2x + 3y - 12 = 0$
47. $x + 3y + 5 = 0$
48. $x - y - 4 = 0$
49. $\lambda = -\frac{2}{7}$
50. 9 sq. units.

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)
21. (a)	**22.** (d)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (c)

Level 2

31. (d)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (b)
41. (a)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (b)	**50.** (d)

Level 3

51. (c)	**52.** (d)	**53.** (b)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (a)	**59.** (b)	**60.** (c)

CONCEPT APPLICATION

Level 1

3. Use slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
4. Use the mid-point formula.
5. Slope $(m) = \tan\theta$.
6. Assume the point on X-axis as$(a, 0)$ and use the distance formula.
7. X-coordinate of the point represents the distance of the point from Y-axis.
8. Use the distance formula.
9. Use the condition for collinearity.
10. Substitute the given point in the line.
11. Use the centroid formula.
12. Use right triangle properties.
13. Find the slopes of AC and BD.
14. Find the intercepts made on X-axis and Y-axis. Then Area $= \dfrac{1}{2}|ab|$.
15. Slope = $\tan\theta$ and intercept on X-axis = 3.
16. Radius of any circle is constant.
17. $x = -1$ is a vertical line and $y = 4$ is a horizontal line.
18. Use the distance formula.
19. Find slope and use point slope form of the line.
20. Use distance formula.
21. Slope $m = \tan\theta$ and the angle between positive X-axis and Y-axis is 90°.
22. Use $AC^2 = AB^2 + BC^2$.
23. Find the roots and substitute in $\dfrac{x}{a} + \dfrac{y}{b} = 1$.
24. The inclination of a line with the positive X-axis is called the slope.
25. (i) If two lines are parallel, then their slopes are equal.
    (ii) The equation of any line parallel to $ax + by + c = 0$ can be taken as $ax + by + k = 0$.
    (iii) Find the X-intercept of the line and equate it to the given x-intercept then get the value of k.
26. (i) Evaluate the vertices of the triangle and proceed.
    (ii) Find the point, where the line cuts X-axis and Y-axis.
    (iii) These two points and the origin are the vertices of the triangle.
    (iv) Then use the formula centroid
    $$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$
27. Use the section formula.
28. Area of the triangle formed by the line $\dfrac{1}{2}|ab|$, a, b are intercepts of the line.
29. (i) $a + b = 5$ and $\dfrac{1}{2}|ab| = 2$.
    (ii) The equation of a line in the intercept form is
    $$\frac{x}{a} + \frac{y}{b} = 1 \quad \Rightarrow \quad a + b = 5$$
    (iii) Then use the formula to find the area of the triangle formed by a line with coordinate axis, is $A = \dfrac{1}{2}|ab|$. $\Rightarrow$ $|ab| = 4$.
30. (i) Evaluate the point using section formula then proceed.
    (ii) Find the point on the required line that divides the line joining the points in the ratio 2 : 3.
    (iii) If two lines are perpendicular then $m_1 \times m_2 = -1$.
    (iv) Find the slope of the required line.
    (v) Hence find the equation of the line by using slope-point formula.

Level 2

31. (i) Use $y = mx + c$ where $m = \tan\theta$.
    (ii) Slope of the required line is $\tan 45°$.
    (iii) Use the slope and y-intercept form.

32. (i) Use the section formula.

(ii) The ratio in which (x, y) divides the line joining the points (x_1, y_1) and (x_2, y_2) is $-(x - x_1)$: $(x - x_2)$ or $-(y - y_1) : (y - y_2)$. If the ratio is negative, it divides externally otherwise divides internally.

33. (i) Find the slope of the given line.

(ii) Use the concept, if two lines are perpendicular to each other, then $m_1 \times m_2 = -1$.

34. First prove the given triangle is a right triangle. In a right triangle, ortho-centre is the vertex containing right angle.

35. First find the point of intersection(x_1, y_1) of first two lines. The equation of the line parallel to X-axis is $y = y_1$.

36. (i) Find the point diving in the ratio 3 : 1.

(ii) Find the equation of line by using point-slope form.

37. (i) Solve equation (2) and equation (3)

(ii) If three lines are concurrent then the point of intersection of any two lines, always lie on the third line.

38. (i) Find the coordinates of the points P and Q.

(ii) Then find the mid-point of PQ say M.

(iii) Now the required line is perpendicular to PQ and passes through M.

39. (i) Prove the given vertices form a right triangle.

(ii) In a right angled triangle, the vertex containing right angle is the ortho-centre.

40. Clearly the given lines forms a right triangle. In a right triangle circum-centre is the mid-point of hypotenuse and ortho-centre is the vertex containing 90°.

41. Find the equation of the line passing through A and mid-point of BC.

42. The area of triangle formed by $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}|ab|$.

43. (i) ABC forms an equilateral triangle.

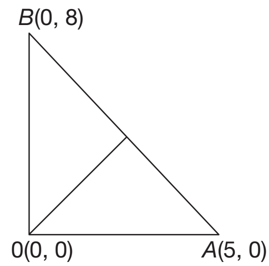
(ii) In an equilateral triangle all the geometric centres except ex-centre coincide.

44. Prove that the given lines form a right triangle, then find the ends of the hypotenuse there by find the mid-point of them.

45. If a line is parallel to X-axis, then its x-coefficient is zero.

46. The vertices of the triangle formed by $8x + 5y = 40$ with the coordinate axes are $0(0, 0)$, $A(5, 0)$, $B(0, 8)$. Mid-point of AB is $\left(\frac{5}{2}, 4\right)$.

Required median is in the form of $y = mx$ and it is passing through $\left(\frac{5}{2}, 4\right)$.



$$\Rightarrow \quad 4 = m\left(\frac{5}{2}\right) \quad \Rightarrow \quad m = \frac{8}{5}$$

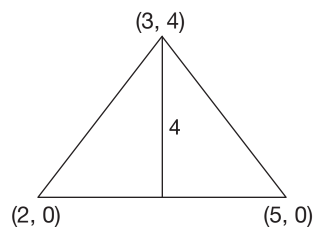
$$\therefore \quad y = \frac{8}{5}x \quad \Rightarrow \quad 8x - 5y = 0.$$

47. $4x - y - 8 = 0$ and $2x + y - 10 = 0$ intersect at (3, 4) and intersect $y = 0$ at (2, 0) and (5, 0) respectively

Since the base is X-axis.

Its height = 4 units

$$\Rightarrow \quad \text{Area} = \frac{1}{2} \times (5 - 2) \times 4 = 6 \text{ sq. units}$$



48. $3x + 4y = 12$

$$\Rightarrow \quad \frac{x}{4} + \frac{y}{3} = 1$$

Vertices of the triangle formed are (4, 0), (0, 3) and (0, 0)

$\Rightarrow$ the longest side is the hypotenuse joining (4, 0) and (0, 3)

$\therefore$ Its length = 5 units.

49. CBDA are in sequential order from first to last.

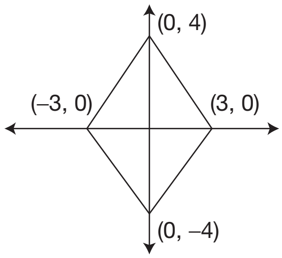
50. BCA are in sequential order from first to last.

Level 3

56. $4\,|x| + 3\,|y| = 12 \quad \Rightarrow \quad 4x - 3y = 12.$

$4x + 3y = 12 - 4x - 3y = 12 - 4x + 3y = 12.$

These lines form a rhombus



$$\text{Area} = \frac{1}{2}d_1d_2 = \frac{1}{2}\times 6\times 8 = 24 \text{ sq. units.}$$

57. The circle is passing through (0, 0), (0, 4), (4, 0)

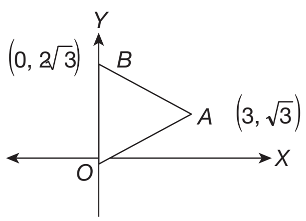
$\Rightarrow$ The circle is the circum-circle of a right triangle whose vertices are (0, 4), (4, 0) and (0, 0)

$\therefore$ Length of the diameter

$$\Rightarrow \quad \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$\text{Radius} = \frac{\sqrt{32}}{2} = \sqrt{8}.$$

58. Let $O(0, 0)$, $A(3, \sqrt{3})$, $B(0, 2\sqrt{3})$



$$OB = \sqrt{(2\sqrt{3})^2} = \sqrt{12}$$

$$AO = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{12}$$

$$AB = \sqrt{3^2 + (\sqrt{3} - 2\sqrt{3})^2} = \sqrt{12}$$

$\therefore$ ΔABC is an equilateral triangle

$\Rightarrow \left(\dfrac{0+3+0}{3}, \dfrac{0+\sqrt{3}+2\sqrt{3}}{3}\right) = (1, \sqrt{3})$ is the centroid as well as circum-centre.

59. $3x - 4y = 10$ (1)

$8x + 6y = 15$

$$4x + 3y = \frac{15}{2}$$

Eqs. (1), (2) are perpendicular lines their intersecting point is the ortho-centre.

On solving Eqs. (1) and (2) we get the ortho-centre as $\left(\dfrac{12}{5}, \dfrac{-7}{10}\right)$.

60. Let the equation of the line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$$\Rightarrow \quad \frac{2}{a} + \frac{3}{b} = 1 \quad \Rightarrow \quad 2b + 3a = ab$$

$$\Rightarrow \quad 3a + 2b = 27$$

$$ab = 27$$

$$b = \frac{27}{a}$$

$$\Rightarrow \quad 3a + 2\times\frac{27}{a} = 27$$

$$3a^2 + 54 = 27a$$

$$3a^2 - 27a + 54 = 0$$

$$3a^2 - 27a + 54 = 0$$

$$a^2 - 9a + 18 = 0$$

$$a^2 - 6a - 3a + 18 = 0$$

$$a(a - 6) - 3(a - 6) = 0$$

$$\Rightarrow \quad a = 6, 3 \quad \Rightarrow \quad b = \frac{9}{2} \quad \text{or} \quad 9$$

Required equation is $\dfrac{x}{6} + \dfrac{y}{\left(\dfrac{9}{2}\right)} = 1$

$$\Rightarrow \quad \frac{3x + 4y}{18} = 1 \quad 3x + 4y = 18$$

or $\dfrac{x}{3} + \dfrac{y}{9} = 1$

$$\Rightarrow \quad 3x + y = 9.$$