Sample Question Paper - 45
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time : 2 Hr. | Max. Marks : 40

General Instructions :

1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. If α and β are the roots of the equation $x^2 + px + q = 0$, then what is value of $\alpha^2 + \beta^2$?

OR

For which value of k does the pair of equation $x^2 - y^2 = 0$ and $(x - k)^2 + y^2 = 1$ yield a unique positive solution of x ?

2. Find how many integers between 200 and 500 are divisible by 8.

3. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3.

4. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

5. For the following distribution :

Below	10	20	30	40	50	60
Number of students	3	12	27	57	75	80

Find the modal class.

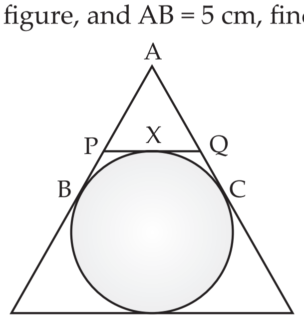
6. The sum of two numbers is 15 and their reciprocals is $\frac{3}{10}$ · Find the numbers.

Section - B

7. The marks attained by 40 students in a short assessment is given below where a and b are missing. If the mean of the distribution is 7.2, find a and b.

Marks	5	6	7	8	9
No. of students	6	a	16	13	b

8. If AB, AC, PQ are the tangents is the figure, and AB = 5 cm, find the perimeter of ΔAPQ.



9. The following table provides data about the weekly wages (in ₹) of workers in a factory. Calculate the Mean and the Modal Class.

Class Interval	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80	80 – 85	85 – 90
Weekly wages (₹)	5	20	10	10	9	6	12	8

10. A kite is flying at a height of 30 m from the ground. The length of the string from the kite to the ground is 60 m. Assuming that there is no slack in the string, then find the angle of elevation of the kite at the ground.

OR

A tower stands vertically on the ground. From a point on the ground which is 25 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 45°. Then find the height (in meters) of the tower.

Section - C

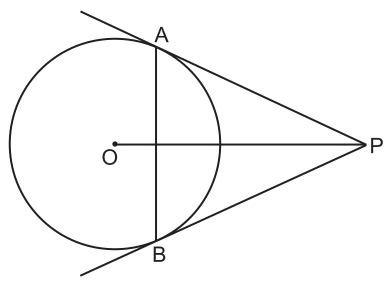
11. A hemispherical depression is cut out from one face of a cubical block of side 7 cm such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

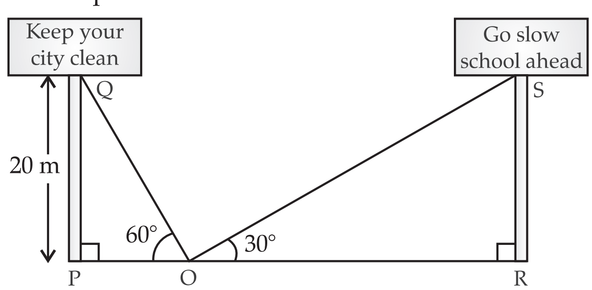
OR

Sushant has a vessel in the shape of an inverted cone that is open at the top. Its height is 11 cm and the radius of the top is 2.5 cm. It is full of water and metallic spherical balls of diameter 0.5 cm are put in the vessel such that $\frac{2}{5}$ th of the water flows out. Find the number of balls that were put in the vessel.

12. In a circle C(O, r), OP is equal to diameter of the circle. PA and PB are two tangents. Prove that ΔABP is an equilateral triangle.



13. Two hoardings are put on two poles of equal heights standing on either side of the road. From a point between them on the road (not the mid point) the angle of elevation of the top of poles are 60° and 30° respectively. Height of the each pole is 20 m.



Answer the following questions. (Take $\sqrt{3}$ = 1.73)

(i) Find the length of PO.

(ii) The width of the road.

14. Aadita is celebrating her birthday. She invited her friends. She bought a packet of toffees/candies. She arranged the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) (a) Find the first term and common difference of A.P.

(b) How many candies are placed in the 9th row?

(ii) (a) Find the difference in number of candies placed in 7th and 3rd row.

(b) Find the number of candies in 12th row.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section - A

1. We know that α and β are the roots of the equation $x^2 + px + q = 0$, $\alpha + \beta = -p$ and $\alpha\beta = q$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2(q)$

$= p^2 - 2q$

OR

$x^2 - y^2 = 0$ ...(i)

and $(x - k)^2 + y^2 = 1$

$x^2 + k^2 - 2kx + y^2 - 1 = 0$ ...(ii)

From equations (i) and (ii)

$2x^2 - 2kx + k^2 - 1 = 0$

For unique solution $b^2 - 4ac = 0$ must satisfy

$(-2k)^2 - 4 \times 2 \times (k^2 - 1) = 0$

$\Rightarrow$ $4k^2 = 8$

$\Rightarrow$ $k = \sqrt{2}$

2. Smallest number divisible by 8 in the given range = 208

Last number divisible by 8 in the given range = 496

So, $a = 208$, $d = 8$, $n = ?$, $T_n = 496$

We know, $T_n = a + (n - 1)d$

$\Rightarrow$ $496 = 208 + (n - 1)\,8$

or $8n + 208 - 8 = 496$

or $8n = 496 - 200$

$\Rightarrow$ $8n = 296$

$\Rightarrow$ $n = \frac{296}{8} = 37$

So number of terms between 200 and 500 divisible by 8 are 37.

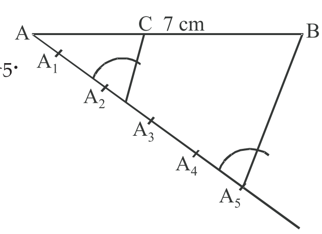
3. Step I : Draw AB = 7 cm

Step II : At A, draw an acute angle with 5 equidistant marks A_1, A_2, A_3, A_4, A_5.

Step III : Join A_5B

Step IV : Draw $A_2C \parallel A_5B$ to get point C on AB.

Step V : Thus, AC : CB = 2 : 3.



4. Let radius of hemisphere be r units.

Volume of hemisphere = S.A. of hemisphere (Given)

$\frac{2}{3}\pi r^3 = 3\pi r^2$

$\Rightarrow$ $r = \frac{9}{2}$ or diameter = 9 units

5.

Below	Class interval	Cumulative frequency	Frequency
10	0 – 10	3	3
20	10 – 20	12	9
30	20 – 30	27	15
40	30 – 40	57	30
50	40 – 50	75	18
60	50 – 60	80	5

Here, modal class is 30 – 40, with maximum frequency 30.

6. Let the numbers be x and $(15 - x)$. So, their reciprocals are $\frac{1}{x}$ and $\frac{1}{15-x}$ respectivley.

Now, $$\frac{1}{x}+\frac{1}{15-x} = \frac{3}{10}$$

$\Rightarrow$ $$\frac{15-x+x}{x(15-x)} = \frac{3}{10}$$

$\Rightarrow$ $$\frac{5}{x(15-x)} = \frac{1}{10}$$

$\Rightarrow$ $$x(15 - x) = 50$$

$\Rightarrow$ $$15x - x^2 = 50$$

$\Rightarrow$ $$x^2 - 15x + 50 = 0$$

$\Rightarrow$ $$x^2 - 10x - 5x + 50 = 0$$

$\Rightarrow$ $$x(x - 10) - 5(x - 10) = 0$$

$\Rightarrow$ $$(x - 5)(x - 10) = 0$$

$\Rightarrow$ $$x = 5 \text{ and } 10.$$

$\therefore$ Numbers are 5 and 10.

Section - B

7.

Class Interval (x_i)	Frequency (f_i)	($f_i \times x_i$)
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
	$\Sigma f_i = 35 + a + b = 40$	$\Sigma(f_i \times x_i) = 246 + 6a + 9b$

We know that, $$\text{Mean} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i}$$

$$7.2 = \frac{246+6a+9b}{40}$$

$\Rightarrow$ $$246 + 6a + 9b = 40(7.2)$$

$\Rightarrow$ $$246 + 6a + 9b = 288$$

$\Rightarrow$ $$6a + 9b = 42$$

$\Rightarrow$ $$2a + 3b = 14 \quad \text{...(i)}$$

Also $$35 + a + b = 40$$

$\Rightarrow$ $$a + b = 5$$

$\Rightarrow \quad 2a + 2b = 10 \quad \ldots\text{(ii)}$

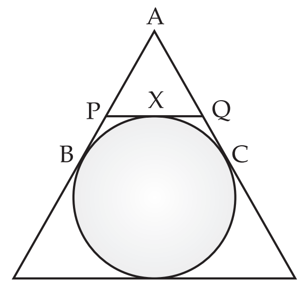
Subtracting equation (ii) from equation (i),

$$b = 4$$

and $\quad a = 5 - 4 = 1.$

8. Given, AB, AC, PQ are tangents and AB = 5 cm

Perimeter of ΔAPQ,



$$\text{Perimeter} = AP + AQ + PQ$$
$$= AP + AQ + (PX + QX)$$

We know that, the two tangents drawn from external point to the circle are equal in length from point A,

So, $\quad AB = AC = 5$ cm

From point P, PX = PB [Since, tangents from an external point to the circle are equal.]

From point Q, QX = QC [Since, tangents from an external point to the circle are equal.]

$\therefore \quad \text{Perimeter} = AP + AQ + PQ$
$$= AP + AQ + (PX + XQ)$$
$$= AP + AQ + (PB + QC)$$
$$= (AP + PB) + (AQ + QC)$$
$$= AB + AC = 5 + 5$$
$$= 10 \text{ cm.}$$

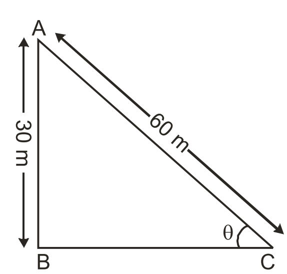
9.

Class Interval	Frequency (f_i)	Class Mark (x_i)	($f_i \times x_i$)
50 – 55	5	52.5	262.5
55 – 60	20	57.5	1150
60 – 65	10	62.5	625
65 – 70	10	67.5	675
70 – 75	9	72.5	652.5
75 – 80	6	77.5	465
80 – 85	12	82.5	990
85 – 90	8	87.5	700
	$N = \Sigma f_i = 80$		$\Sigma(f_i \times x_i) = 5520$

Thus $\quad \text{Mean} = \dfrac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \dfrac{5520}{80} = ₹\ 69$

Since 55 – 60 has the highest frequency 20, so it is the Modal Class.

10. Given, perpendicular height = 30 m and hypotenuse = 60 m.



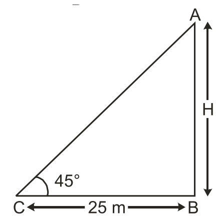
We know, $\quad \sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$= \frac{30}{60} = \frac{1}{2} = \sin 30°$$

Thus, $\quad \theta = 30°.$

OR

Let the height of the tower be H m.

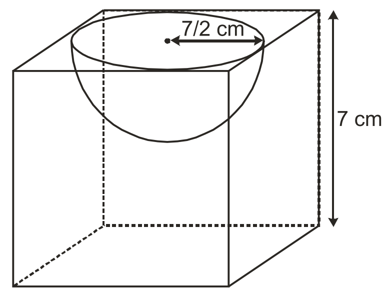


Thus, $\tan 45° = \dfrac{\text{Perpendicular}}{\text{Base}}$

or $\dfrac{H}{25} = 1$

Hence $H = 25$ m.

Section - C



11. Given, Side of the cube = 7 cm

Diameter of the hemisphere = 7 cm

Thus, Radius of the hemisphere = 3.5 cm

Total surface area of the cube = $6(7)^2$ cm^2

Curved surface area of the hemisphere = $2\pi(3.5)^2$ cm^2

Thus, total surface area of the remaining solid

= Total surface area of cube

+ Curved surface area of the hemisphere

– Area of the top face of the hemisphere

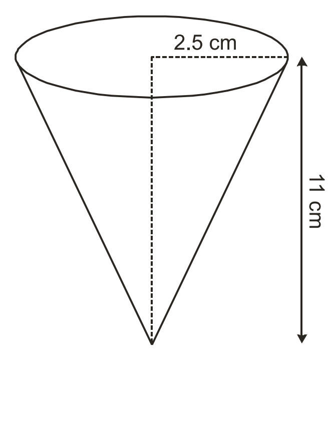
$= [6(7)^2 + 2\pi(3.5)^2 - \pi(3.5)^2]$ cm^2

$= \left[6(7)^2 + \left(\dfrac{22}{7}\right)(3.5)^2\right]$cm^2

$= \left[294 + \dfrac{77}{2}\right]$cm^2

= 332.5 cm^2.

OR



Given, Height of the cone = 11 cm

Radius of the top of the cone = 2.5 cm

Diameter of each ball = 0.5 cm

Thus, Radius of each ball = 0.25 cm

Volume of water that flows out = $\dfrac{2}{5}$ (Volume of the cone)

Now, Volume of the water in the cone = $\dfrac{1}{3}\pi r^2 h$

$= \dfrac{\pi}{3}(2.5)^2\,(11)$ cm^3

$= \dfrac{11\pi}{3}(2.5 \times 2.5)$ cm^3

Thus, the volume of water that flows out

$= \dfrac{11}{3}\pi(2.5 \times 2.5)\left(\dfrac{2}{5}\right)$cm^3

= Total volume of all the spherical balls

Now, volume of 1 spherical ball $= \left(\dfrac{4}{3}\right)\pi(0.25)^3$ cm^3

$= \left(\dfrac{4}{3}\right)\pi(0.25 \times 0.25 \times 0.25)$ cm^3

Hence, the number of spherical balls $= \dfrac{\left(\frac{1}{3}\right)11\pi(2.5\times2.5)\left(\frac{2}{5}\right)}{\left(\frac{4}{3}\right)\pi(0.25\times0.25\times0.25)}$

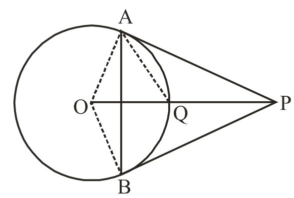
$$= \frac{11(2.5\times2.5)}{10(0.25\times0.25\times0.25)}$$

$$= 440$$

Thus, number of balls = 440.

12. Let OP meet the circle at Q

It is given that,



$$OP = \text{Diameter} = 2r$$

$\Rightarrow$ $OP = OQ + QP$

$\Rightarrow$ $2r = r + QP$

$\Rightarrow$ $QP = r$

$\therefore$ Q is the mid-point of OP.

Now AP is a tangent at A and OA is radius

$\Rightarrow$ $OA \perp AP$

$\therefore$ ΔOAP is right triangle with A = 90° and Q is mid-point of hypotenuse OP.

$\therefore$ $AQ = OQ = OA$

$\therefore$ ΔOAQ is equilateral.

$\Rightarrow$ $\angle AOQ = 60°$

$\therefore$ $\angle APO = 180° - 90° - 60° = 30°$

$\therefore$ $\angle APB = 2\,\angle APO$

$= 2 \times 30° = 60°$

Also, $PA = PB$

$\Rightarrow$ $\angle PAB = \angle PBA$

Now in ΔAPB, we have

$$\angle APB + \angle PAB + \angle PBA = 180°$$

$\Rightarrow$ $60° + 2\,\angle PAB = 180°$

$\Rightarrow$ $\angle PAB = 60°$

$\therefore$ $\angle PAB = \angle PBA = \angle APB = 60°$

$\therefore$ ΔAPB is equilateral.

13. (i) In ΔOPQ, we have

$$\tan 60° = \frac{PQ}{PO}$$

$\Rightarrow$ $\sqrt{3} = \dfrac{20}{PO}$

$\Rightarrow$ $PO = \dfrac{20}{\sqrt{3}}\text{ m} = 11.56 \text{ m}$

(ii) $\because$ $\tan 30° = \dfrac{SR}{OR}$

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{20}{OR}$

$\Rightarrow$ $OR = 20\sqrt{3}\text{ m}$

Clearly, width of road = PR

$$= PO + OR = \left(\frac{20}{\sqrt{3}} + 20\sqrt{3}\right)\text{m}$$

$$= 20\left(\frac{4}{\sqrt{3}}\right)\text{m}$$

$$= \frac{80}{\sqrt{3}}\text{m} = 42.24 \text{ m}$$

14. (i) (a) As A.P. = 3, 5, 7 ...

$\therefore$ $a = 3, d = 2$

(b) Since, $a_n = a + (n - 1)d$

$a_9 = 3 + 8 \times 2 = 19$

(ii) (a) $a_7 - a_3 = a + 6d - a - 2d = 4d$

$= 4 \times 2 = 8$

(b) Number of candies in 12^{th} row,

$a_{12} = a + 11d$

$= 3 + 11 \times 2 = 25.$