Sample Question Paper - 47
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time : 2 Hr. Max. Marks : 40

General Instructions :

1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. Calculate the value of p from the following data :

Class	Frequency
0 – 20	8
20 – 40	15
40 – 60	p
60 – 80	12
80 – 100	5
	$N = \Sigma f_i = 60$

2. If the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$ are equal, where $b \neq 0$, then find the relation between a, b and c.

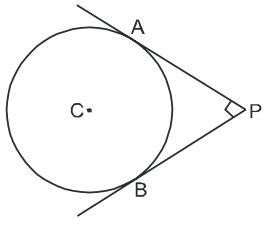
OR

If 3 is a root of the equation $kx^2 - kx - 3 = 0$, find then the value of k

3. The surface area of two spheres are in the ratio 16 : 9. Find the ratios of their volumes.

4. The sum of first 20 odd natural numbers.

5. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA is perpendicular to PB, then find the length of each tangent.



6. If the zeros of the quadratic equation $x^2 + (a + 1)x + b + 1 = 0$ are 2 and – 3, then find the values of a and b.

Section - B

7. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

8. The following tables gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

9. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds, the angle of elevation become 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

OR

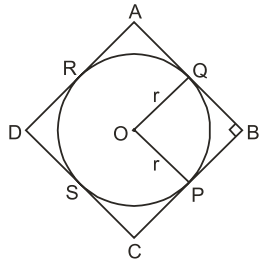
A highway leads to the foot of 300 m high tower. An observatory is set at the top of the tower. It sees a car moving towards it at an angle of depression of 30°. After 15 seconds, angle of depression becomes 60°. Find the distance travelled by the car during this time.

10. Mode of the following frequency distribution is 65 and sum of all the frequencies is 70. Find the missing frequencies x and y.

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140	140 – 160
Frequency	8	11	x	12	y	9	9	5

Section - C

11. In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90°$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the value of radius (r).



12. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

OR

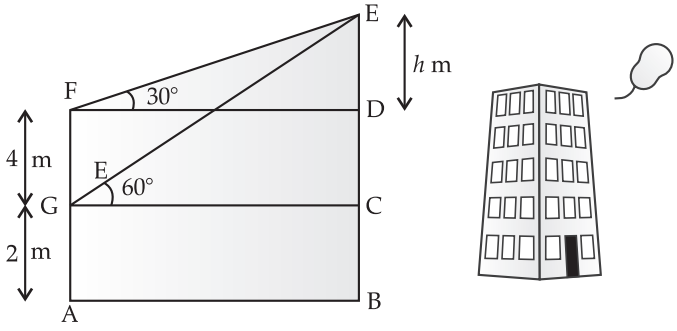
A tent is in the form of a right circular cylinder of base radius 14 m and height 3 m is surmounted by a right circular cone of the same base radius. The total height of the tent is 13.5 m. Find the cost of the canvas used in making the tent at ₹ 80 per square meter and the cost of painting it at ₹ 2 per square meter. $\left[\text{Use } \pi = \frac{22}{7}\right]$

13. Treasure Hunt Games : While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an AP. If the number of the nth spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.

(i) Which number is on the $(n-2)^{th}$ spot ?

(ii) Which spot is numbered as 116 ?

14. There are two windows in a house. First window is at the height of 2 m above the ground and other window is 4 m vertically above the lower window. Ankit and Radha are sitting inside the two windows at point G and F respectively. At an instant, the angle of elevation of a balloon from these windows are observed to be 60° and 30° as shown in the diagram :



Answer the following questions.

(i) Find the value of DF.

(ii) What is the value of h in the given figure ?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section - A

1. Given, $N = \Sigma f_i = 60$

$\Rightarrow 8 + 15 + p + 12 + 5 = 60$

$\Rightarrow 40 + p = 60$

$\Rightarrow p = 20.$

2. $(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$

The given roots are equal, and then D must be zero

$[2(b^2 - ac)]^2 - 4(a^2 - bc)(c^2 - ab) = 0$

$\Rightarrow 4(b^4 + a^2c^2 - 2ab^2c) - 4(a^2c^2 - a^3b - bc^3 + ab^2c) = 0$

$\Rightarrow 4b^4 - 12ab^2c + 4bc^3 + 4a^3b = 0$

$\Rightarrow 4b(b^3 - 3abc + c^3 + a^3) = 0$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

OR

If 3 is the root of $kx^2 - kx - 3 = 0$ then 3 satisfy it *i.e.*,

$9k - 3k - 3 = 0$

$\Rightarrow 6k - 3 = 0$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{3}{6}$

$\Rightarrow k = \frac{1}{2}$

3. Let the radius of the larger sphere be R and the smaller be r.

Given, $4\pi R^2 : 4\pi r^2 = 16 : 9$

$\Rightarrow R^2 : r^2 = 4^2 : 3^2$

$\Rightarrow R : r = 4 : 3$

Thus, $V_1 : V_2 = \frac{4\pi}{3}R^3 : \frac{4\pi}{3}r^3 = 4^3 : 3^3$

or $V_1 : V_2 = 64 : 27$

4. The given A.P. is

$$1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39$$

Thus $a = 1$, $d = 2$ and $n = 20$

We know that, $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$= \frac{20}{2}\{2(1) + (20-1)2\}$$

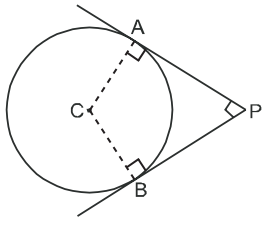
$$= 10\{2 + 38\} = 400.$$

5. Now, AP = BP [Since, tagnents from an external point P are equal]

Join CA and CB and they are perpendicular to AP and AB respectively.

Given CA = CB = 4 cm (Radius of circle)

Now, $\angle ACB = 360° - [90° + 90° + 90°]$



or $\angle ACB = 90°$

As APBC is a right-angled quadrilateral with its adjacent sides CA and CB being 4 cm.

So AP = PB = 4 cm

[Opposite sides to adjacent sides of a right-angled quadrilateral]

6. If 2 and – 3 are the roots of $x^2 + (a+1)x + b + 1 = 0$ then they satisfy this equation.

$$4 + (a + 1)\,2 + b + 1 = 0$$

$$\Rightarrow \quad 4 + 2a + 2 + b + 1 = 0$$

$$\Rightarrow \quad 2a + b + 7 = 0$$

$$\Rightarrow \quad 2a + b = -7 \qquad ...(1)$$

also, $9 - 3\,(a + 1) + b + 1 = 0$

$$9 - 3a - 3 + b + 1 = 0$$

$$-3a + b + 7 = 0$$

$$-3a + b = -7 \qquad ...(2)$$

On subtracting (1) from (2), we get

$$-5a = 0$$

$$\Rightarrow \quad a = 0$$

Putting $a = 0$ in equation (1), we get $b = -7$

Hence a and b are 0 and – 7.

Section - B

7. Steps of constructions :

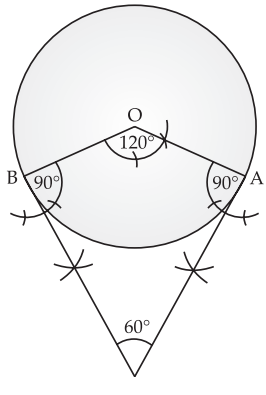
Step I : Draw a circle with O as center and radius 4 cm.

Step II : Draw any $\angle AOB = 120°$ $(180° - 60°)$

Step III : From A and B draw $\angle PAO = \angle PBO = 90°$ which meet at P.

$\therefore$ PA and PB are the required tangents.

Hence $\angle BPA = 60°$, as PBOA is a cyclic quadrilateral.



8.

Litracy rate (in %)	Number of cities (f_i)	Class mark (x_i)	$d_i = x_i - 70$	$f_i d_i$
45 – 55	3	50	– 20	– 60
55 – 65	10	60	– 10	– 100
65 – 75	11	70 = A	0	0
75 – 85	8	80	10	80
85 – 95	3	90	20	60
				$\Sigma f_i d_i = -20$

Here, we have

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 70 - \frac{20}{35}$$

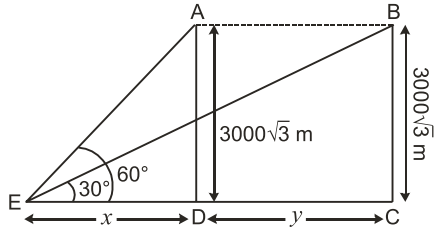
$$= 70 - 0.57 = 69.43$$

Hence, the mean literacy rate is 69.43%.

9. Let the ground distance between the aeroplane A and the point E be x m.

Given, height AD is $3000\sqrt{3}$ m and the angle of elevation is 60°.

So, in ΔAED,



$$\tan 60° = \frac{3000\sqrt{3}}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3000\sqrt{3}}{x}$$

$$\Rightarrow \quad x = 3000 \text{ m}$$

Let the new distance (DC) covered by aeroplane in 30 seconds be y m.

So,
$$\tan 30° = \frac{3000\sqrt{3}}{3000 + y}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{3000 + y}$$

$$\Rightarrow \quad 3000 + y = (3000\sqrt{3})\sqrt{3}$$

$$\Rightarrow \quad 3000 + y = 9000$$

$$\Rightarrow \quad y = 9000 - 3000$$

$$= 6000 \text{ m}$$

Thus, distance covered in 30 seconds = 6000 m

Hence,
$$\text{Speed} = \frac{6000}{30}$$

$$= 200 \text{ m/sec}$$

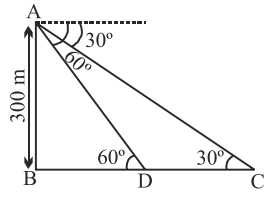
$$= 200 \times \frac{18}{5} \text{ km/hr}$$

Hence, the speed of the aeroplane is 720 km/hr.

OR

Let AB = 300 m is the tower. Initially, car is at C and after 15 seconds, it reaches at D.

In right ΔABC,



$$\frac{AB}{BC} = \tan 30°$$

$$\Rightarrow \quad \frac{300}{BC} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $BC = 300\sqrt{3}$ m

In right ΔABD,

$$\frac{AB}{BD} = \tan 60°$$

$\Rightarrow$ $$\frac{300}{BD} = \sqrt{3}$$

$\Rightarrow$ $$BD = \frac{300}{\sqrt{3}}\text{m}$$

$\therefore$ Distance covered by car $= DC = BC - BD$

$$= 300\sqrt{3} - \frac{300}{\sqrt{3}} = \frac{600}{\sqrt{3}}$$

$$= 200\sqrt{3} \text{ m}$$

10.

Class Interval	Frequency
0 – 20	8
20 – 40	11
40 – 60	$x(f_0)$
60 – 80	$12(f_1)$
80 – 100	$y(f_2)$
100 – 120	9
120 – 140	9
140 – 160	5
	$\Sigma f = 70$

Here, $8 + 11 + x + 12 + y + 9 + 9 + 5 = 70$ [Given]

$\Rightarrow$ $54 + x + y = 70$

$\Rightarrow$ $x + y = 70 - 54 = 16$ ...(i)

$\text{Mode} = 65$ [Given]

$\therefore$ Modal Class is 60 – 80

So, $l = 60$, $h = 20$, $f_0 = x$, $f_1 = 12$, $f_2 = y$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$\therefore$ $$65 = 60 + \frac{12 - x}{2(12) - x - y} \times 20$$

$\Rightarrow$ $$65 - 60 = \frac{12 - x}{24 - (x + y)} \times 20$$

$\Rightarrow$ $$5 = \frac{12 - x}{24 - 16} \times 20$$ [From equation (i)]

$\Rightarrow$ $$5 = \frac{12 - x}{8} \times 20$$

$\Rightarrow$ $$5 \times \frac{2}{5} = 12 - x$$

$$2 = 12 - x$$

$\Rightarrow$ $x = 12 - 2 = 10$

$x + y = 16$ [From equation (i)]

$\Rightarrow$ $10 + y = 16$

$\Rightarrow$ $y = 16 - 10 = 6$

$\therefore$ $x = 10, y = 6$

Section - C

11. Given, $\angle B = 90°$, AD = 23 cm, AB = 29 cm, DS = 5 cm and OQ = OP = r.

Now, DS = DR = 5 cm [Tangents from point D]

Thus, AR = AD – DR

= (23 – 5) cm = 18 cm

Hence, AQ = 18 cm [Tangents from point A]

or BQ = AB – AQ

= (29 – 18) cm = 11 cm

Also, $\angle OQB = \angle OPB = 90°$ [Both the radii are perpendicular to tangents AB and BC respectively]

Hence, in the quadrilateral POQB,

$\angle QOP = 360° – (90° + 90° + 90°)$

$= 360° – 270° = 90°$

In quadrilateral POQB,

$\angle QOP = \angle OPB = \angle PBQ = \angle BQO = 90°$

and BQ = PB [Tangents from point B]

i.e. all angles are 90° and adjacent sides are equal.

Hence, POQB is a square

$\therefore$ OQ = BQ = 11 cm

But OQ = OP [Radii of the same circle]

So, OQ = OP = PB = QB = 11 cm

Thus, the radius of the circle is 11 cm.

12. We have, diameter of well = 4 m

$\therefore$ Radius, r = 2 m

and Height, h = 14 m.

Volume of earth taken out after digging the well

$= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 14$

$= 176 \text{ m}^3$

Let x be the width of the embankment formed by the earth taken out.

Then, volume of embankment

$\Rightarrow \frac{22}{7}[(2+x)^2 - (2)^2] \times \frac{40}{100} = 176$

$\Rightarrow \frac{22}{7}[4 + x^2 + 4x - 4] \times \frac{2}{5} = 176$

$\Rightarrow x^2 + 4x = \frac{176 \times 5 \times 7}{22 \times 2}$

$\Rightarrow x^2 + 4x - 140 = 0$

$\Rightarrow x^2 + 14x - 10x - 140 = 0$

$\Rightarrow x(x + 14) - 10(x + 14) = 0$

$\Rightarrow (x + 14)(x - 10) = 0$

$\Rightarrow x = -14 \text{ or } 10$

$x = -14$(neglect)

$\therefore x = 10$

Hence, width of embankment = 10 m.

OR

Given, height of the cylindrical structure = 3 m

Total height of the tent = 13.5 m

Base radius of cylindrical structure = Base radius of conical structure = 14 m

Rate of canvas = ₹ 80 per m^2

Rate of painting = ₹ 2 per m^2

Now, height of the conical structure = (13.5 – 3) m = 10.5 m

∴ Total surface area of the tent = Curved surface area of the (cylindrical structure + conical structure)

$= 2\pi rh + \pi rl$

$= \pi r\,(2h + l)$

$= \pi(14)[2(3) + (\sqrt{(14)^2 + (10.5)^2}\,]\ m^2$

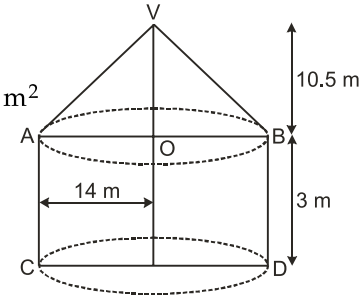
$= \pi(14)(6 + \sqrt{196 + 110.25})\ m^2$

$= (14)\pi(6 + \sqrt{306.25})\ m^2$

$= (14)\frac{22}{7}(6 + \sqrt{(17.5)^2})\ m^2$

$= 44(6 + 17.5)\ m^2$

$= 44 \times 23.5\ m^2 = 1034\ m^2$



Thus, the cost of canvas = ₹ (80 × 1034)

= ₹ 82720

and, the cost of painting = ₹ (2 × 1034)

= ₹ 2068.

13. (i) Number on $(n-2)^{th}$ spot $= t_{n-2} = 20 + 4(n-2)$

$= 20 + 4n - 8 = 12 + 4n.$

(ii) Let n^{th} spot be numbered as 116.

∴ $t_n = 116$

⇒ $20 + 4n = 116$

⇒ $4n = 96$

⇒ $n = 24.$

14. (i) In ΔEFD,

$$\tan 30° = \frac{ED}{DF}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{DF}$$

$$\Rightarrow \quad DF = h\sqrt{3}\ m$$

(ii) In ΔGCE,

$$\tan 60° = \frac{EC}{GC} = \frac{h+4}{DF} \qquad (\because GC = DF)$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+4}{\sqrt{3}h} \qquad (\because DF = h\sqrt{3})$$

$$\Rightarrow \quad 3h = h + 4$$

$$\Rightarrow \quad h = 2.$$