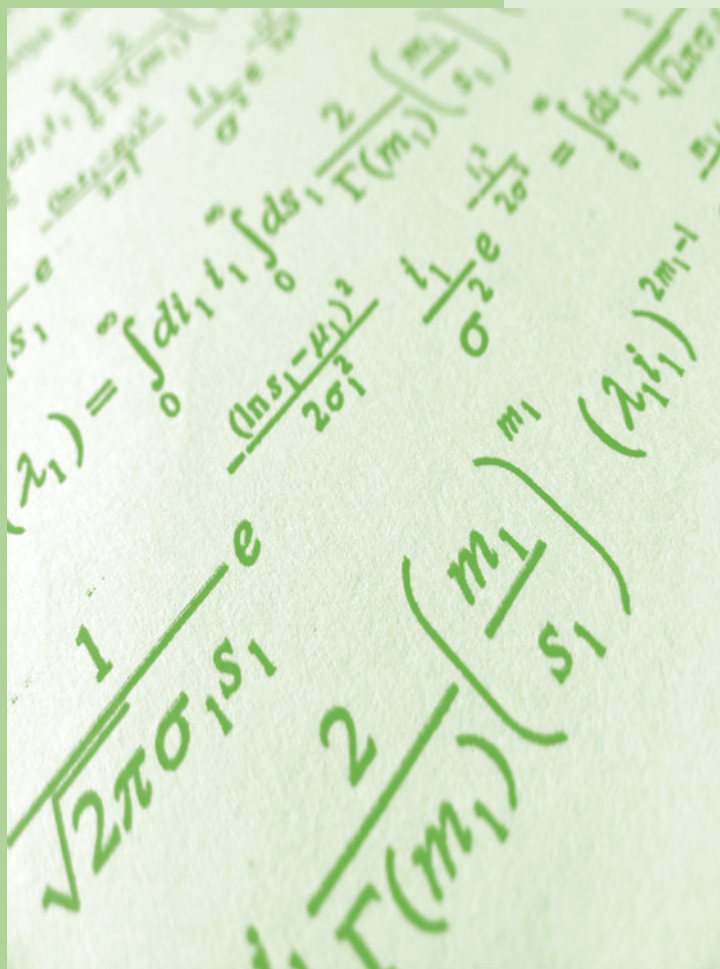


Chapter 3

Linear Equations in Two Variables



REMEMBER

Before beginning this chapter, you should be able to:

- Solve simple equations with one/two variable
- Use graphs of linear equations and in-equations
- Solve basic word problems on linear equations and in-equations

KEY IDEAS

After completing this chapter, you would be able to:

- Solve linear equations by different methods
- Learn nature of solutions of simultaneous linear equations
- Study solving of word problems on linear equations with two variables
- Find solution of linear equations by graphical methods

INTRODUCTION

While solving the problems, in most cases, first we need to frame an equation. In this chapter, we will learn how to frame and solve equations. Framing an equation is more difficult than solving an equation. Now, let us review the basic concepts related to this chapter.

ALGEBRAIC EXPRESSIONS

Expressions of the form $2x$, $(3x + 5)$, $(4x - 2y)$, $2x^2 + 3\sqrt{y}$, $\frac{3x^2}{2}\sqrt{y}$ are algebraic expressions.

$3x$ and 5 are the terms of $(3x + 5)$ and $4x$ and $2y$ are the terms of $4x - 2y$. Algebraic expressions are made of numbers, symbols and the basic arithmetical operations. In the term $2x$, 2 is the numerical coefficient of x and x is the variable coefficient of 2 .

EQUATION

An equation is a sentence in which there is an equality sign between two algebraic expressions.

For example, $2x + 5 = x + 3$, $3y - 4 = 20$ and $5x + 6 = x + 1$ are some examples of equations. Here x and y are unknown quantities and 5 , 3 , 20 , etc., are known quantities.

Linear Equation

An equation, in which the highest index of the unknowns present is one, is a linear equation.

$2(x + 5) = 18$, $3x - 2 = 5$, $x + y = 20$ and $3x - 2y = 5$ are some linear equations.

Simple Equation

A linear equation which has only one unknown is a simple equation. $3x + 4 = 16$ and $2x - 5 = x + 3$ are examples of simple linear equations.

The part of an equation which is to the left side of the equality sign is known as the left hand side, abbreviated as LHS. The part of an equation which is to the right side of the equality sign is known as the right hand side, abbreviated as RHS. The process of finding the value of an unknown in an equation is called solving the equation. The value/values of the unknown found after solving an equation is/are called the solutions or the roots of the equation.

Before we learn how to solve an equation, let us review the basic properties of equality.

1. When a term is added to both sides of the equality, the equality does not change.

Example: If $a + b = c + d$, then $a + b + x = c + d + x$.

This property holds good for subtraction also.

2. When the expressions on the LHS and RHS of the equation are multiplied by a non-zero term, the equation does not change.

Example: If $a + b = c + d$, then $x(a + b) = x(c + d)$.

This property holds good for division also.

Solving an Equation in One Variable

The following steps are involved in solving an equation.

Step 1: Always ensure that the unknown quantities are on the LHS and the known quantities or constants on the RHS.

Step 2: Add all the terms containing the unknowns on the LHS and all the knowns on the RHS, so that each side of the equation contains only one term.

Step 3: Divide both sides of the equation by the coefficient of the unknown.

EXAMPLE 3.1

If $4x + 15 = 35$, then find the value of x .

SOLUTION

Step 1: Group the known quantities as the RHS of the equation, i.e., $4x = 35 - 15$.

Step 2: Simplify the numbers on the RHS $\Rightarrow 4x = 20$.

Step 3: Since 4 is the coefficient of x , divide both the sides of the equation by 4.

$$\frac{4x}{4} = \frac{20}{4} \Rightarrow x = 5.$$

EXAMPLE 3.2

Solve: $3x + 11 = 6x - 13$

SOLUTION

Step 1: $6x - 3x = 11 + 13$

Step 2: $3x = 24$

Step 3: $\frac{3x}{3} = \frac{24}{3} \Rightarrow x = 8.$

EXAMPLE 3.3

A swarm of 62 bees flies in a garden. If 3 bees land on each flower, 8 bees are left with no flowers. Find the number of flowers in the garden.

SOLUTION

Let the number of flowers be x .

Number of bees on the flowers = $3x$

Total number of bees = 62

$$\therefore 3x + 8 = 62$$

$$\Rightarrow 3x = 62 - 8 = 54 \Rightarrow x = 18$$

$\therefore$ There are 18 flowers in the garden.

Transposition

In the above problem, $3x + 8 = 62$ can be written as $3x = 62 - 8$. When a term is moved (transposed) from one side of the equation to the other side, the sign is changed. The positive sign is changed to the negative sign and multiplication is changed to division. Moving a term from one side of the equation to the other side is called transposition. Thus solving a linear equation, in general, comprises two kinds of transposition.

Simultaneous Linear Equations

We have learnt to solve an equation with one unknown. Very often we come across equations involving more than one unknown. In such cases we require more than one condition or equation. Generally, when there are two unknowns, we require two equations to solve the problem. When there are three unknowns, we require three equations and so on.

We need to find the values of the unknowns that satisfy all the given equations. Since the values satisfy all the given equations we call them simultaneous equations. In this chapter, we deal with simultaneous (linear) equations in two unknowns.

Let us consider the equation, $2x + 5y = 19$, which contains two unknown quantities x and y .

Here, $5y = 19 - 2x$

$$\Rightarrow y = \frac{19 - 2x}{5} \quad (1)$$

In the above equation for every value of x , there exists a corresponding value for y .

When $x = 1$, $y = \frac{17}{5}$

When $x = 2$, $y = 3$ and so on.

If there is another equation, of the same kind, say, $5x - 2y = 4$.

From this, we get,

$$y = \frac{5x - 4}{2} \quad (2)$$

If we need the values of x and y such that both the equations are satisfied, then $\frac{19 - 2x}{5} = \frac{5x - 4}{2}$

$$\Rightarrow 38 - 4x = 25x - 20$$

$$\Rightarrow 29x = 58$$

$$\Rightarrow x = 2$$

On substituting the value of $x = 2$ in Eq. (1), we get,

$$y = \frac{19 - 2(2)}{5} = \frac{15}{5} = 3$$

$$\Rightarrow y = 3.$$

Both the equations are satisfied by the same values of x and y . Thus we can say that when two or more equations are satisfied by the same values of unknown quantities, then those equations are called simultaneous equations.

When two equations, each in two variables, are given, they can be solved in five ways.

1. Elimination by cancellation
2. Elimination by substitution
3. Adding the two equations and subtracting one equation from the other
4. Cross-multiplication method
5. Graphical method

Elimination by Cancellation

EXAMPLE 3.4

If $2x + 3y = 19$ and $5x + 4y = 37$, then find the values of x and y .

SOLUTION

In this method, the two equations are reduced to a single variable equation by eliminating one of the variables.

Step 1: Here, let us eliminate the y term, and in order to eliminate the y term, we have to multiply the first equation with the coefficient of y in the second equation and the second equation with the coefficient of y in the first equation so that the coefficients of y terms in both the equations become equal.

$$(2x + 3y = 19)4 \Rightarrow 8x + 12y = 76 \quad (1)$$

$$(5x + 4y = 37)3 \Rightarrow 15x + 12y = 111 \quad (2)$$

Step 2: Subtract Eq. (2) from Eq. (1),

$$(15x + 12y) - (8x + 12y) = 111 - 76$$

$$\Rightarrow 7x = 35$$

$$\Rightarrow x = 5.$$

Step 3: Substitute the value of x in Eq. (1) or Eq. (2) to find the value of y . Substituting the value of x in the first equation, we get,

$$2(5) + 3y = 19$$

$$\Rightarrow 3y = 19 - 10 \Rightarrow 3y = 9$$

$$\Rightarrow y = 3.$$

$\therefore$ The solution of the given pair of equation is $x = 5$; $y = 3$.

Elimination by Substitution

EXAMPLE 3.5

If $4x - 3y = 32$ and $x + y = 1$, then find the values of x and y .

SOLUTION

In this method, the two equations are reduced to a single variable equation by substituting the value of one variable, obtained from one equation, in the other equation.

Step 1: Using the second equation, find x in terms of y , i.e.,

$$\begin{aligned}x + y &= 1 \\ \Rightarrow y &= 1 - x\end{aligned}\tag{1}$$

Step 2: Substitute the value of y in the first equation to find the value of x .

$$\therefore 4x - 3(1 - x) = 32$$

Step 3: Simplify the equation in terms of x and find the value of x .

$$\begin{aligned}4x - 3 + 3x &= 32 \\ \Rightarrow 7x &= 32 + 3 = 35 \\ \Rightarrow x &= 5\end{aligned}$$

Step 4: Substituting the value of x in Eq. (1), we have,

$$\begin{aligned}y &= 1 - x = 1 - 5 \\ \Rightarrow y &= -4.\end{aligned}$$

$\therefore$ The solution for the given pair of equations is $x = 5$; $y = -4$.

Adding Two Equations and Subtracting One Equation from the Other

EXAMPLE 3.6

Solve $3x + 7y = 32$ and $7x + 3y = 48$.

SOLUTION

Given,

$$3x + 7y = 32\tag{1}$$

$$7x + 3y = 48\tag{2}$$

Step 1: Adding both the equations, we get

$$\begin{aligned}10x + 10y &= 80 \\ \Rightarrow 10(x + y) &= 10 \times 8 \\ \Rightarrow x + y &= 8.\end{aligned}\tag{3}$$

Step 2: Subtracting Eq. (1) from Eq. (2),

$$\begin{aligned}(7x + 3y) - (3x + 7y) &= 48 - 32 \\ \Rightarrow 4x - 4y &= 16 \\ \Rightarrow 4(x - y) &= 4 \times 4 \\ \Rightarrow x - y &= 4.\end{aligned}\tag{4}$$

Step 3: Adding the Eqs. (3) and (4),

$$\begin{aligned}x + y + x - y &= 12 \\ \Rightarrow 2x &= 12 \\ \Rightarrow x &= 6.\end{aligned}$$

Substituting $x = 6$ in any of the Eqs. (1), (2), (3) or (4), we get, $y = 2$.

$\therefore$ The solution of the pair of equations is $x = 6$; $y = 2$.

Cross-multiplication Method

EXAMPLE 3.7

Solve $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

SOLUTION

Given,

$$a_1x + b_1y + c_1 = 0 \quad (1)$$

$$a_2x + b_2y + c_2 = 0 \quad (2)$$

Solving the above equations using elimination by cancellation method, we get,

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}.$$

Applying alternendo on the above ratios, we have

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1} \text{ and } \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}.$$

The above result can be better remembered using the Fig. 3.1.

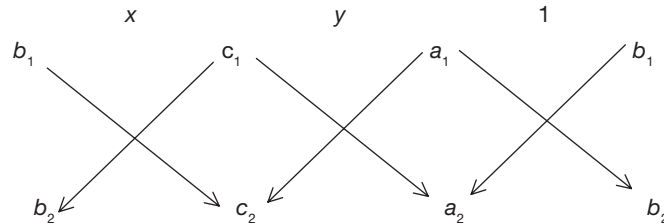


Figure 3.1

The arrows between the two numbers indicate that they are to be multiplied and second product is to be subtracted from the first.

While using this method, the following steps are to be followed.

Step 1: Write the given equations in the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Step 2: Write the coefficients of the equations as mentioned above.

Step 3: Find the values of x and y .

EXAMPLE 3.8

Solve $4x + 5y = 71$ and $5x + 3y = 66$.

SOLUTION

Step 1: Rewriting the equations, we get,

$$4x + 5y - 71 = 0 \quad (1)$$

$$5x + 3y - 66 = 0 \quad (2)$$

Step 2: Write the coefficients of x and y in the specified manner.

$$\begin{array}{rrrr} x & y & & 1 \\ 5 & -71 & 4 & 5 \\ 3 & -66 & 5 & 3 \end{array}$$

Step 3: Find the values of x and y .

$$\begin{aligned} \frac{x}{-330 + 213} &= \frac{y}{-355 + 264} = \frac{1}{12 - 25} \\ \Rightarrow \frac{x}{-117} &= \frac{y}{-91} = \frac{1}{-13} \\ \Rightarrow x &= \frac{-117}{-13}; y = \frac{-91}{-13} \\ \Rightarrow x &= 9 \text{ and } y = 7. \end{aligned}$$

Note Choosing a particular method to solve a pair of equations makes the simplification easier. One can learn as to which method is the easiest to solve a pair of equations by becoming familiar with the different methods of solving the equation.

Graphical Method

Plotting the Points If we consider any point in a plane, then we can determine the location of the given point, i.e., we can determine the distance of the given point from X -axis and Y -axis. Therefore, each point in the plane represents the distance from both the axes. So, each point is represented by an ordered pair and it consists of x -coordinate and y -coordinate. The first element of an ordered pair is called x -coordinate and the second element of an ordered pair is called y -coordinate. In the first quadrant Q_1 , both the x -coordinate and y -coordinate are positive real numbers. In the second quadrant Q_2 , x -coordinates are negative real numbers and y -coordinates are positive real numbers. In the third quadrant Q_3 both the x -coordinate and y -coordinate are negative real numbers. In the fourth quadrant Q_4 , x -coordinates are positive real numbers and y -coordinates are negative real numbers. And the origin is represented by $(0, 0)$.

Consider the point $(2, 3)$. Here 2 is the x -coordinate and 3 is the y -coordinate. The point $(2, 3)$ is 2 units away from the Y -axis and 3 units away from the X -axis. The point $(2, 3)$ belongs to the first quadrant. If we consider the point $(-3, -5)$, -3 is x -coordinate and -5 is y -coordinate. The point $(-3, -5)$ belongs to Q_3 and is 3 units away from the Y -axis and 5 units away from the X -axis.

To plot a point say $P(-3, 4)$, we start from the origin and proceed 3 units towards the left hand side along the X -axis (i.e., negative direction as x -coordinate is negative), and from there we move 4 units upwards along the Y -axis (i.e., positive direction as y -coordinate is positive). The method of plotting a point in a coordinate plane was explained by Rene Des Cartes, a French mathematician.

EXAMPLE 3.9

Plot the following points on the coordinate plane.

$P(2, 3)$, $Q(-4, -5)$, $R(2, -3)$, $S(-4, 4)$, $T(3, 0)$, $U(0, 5)$.

SOLUTION

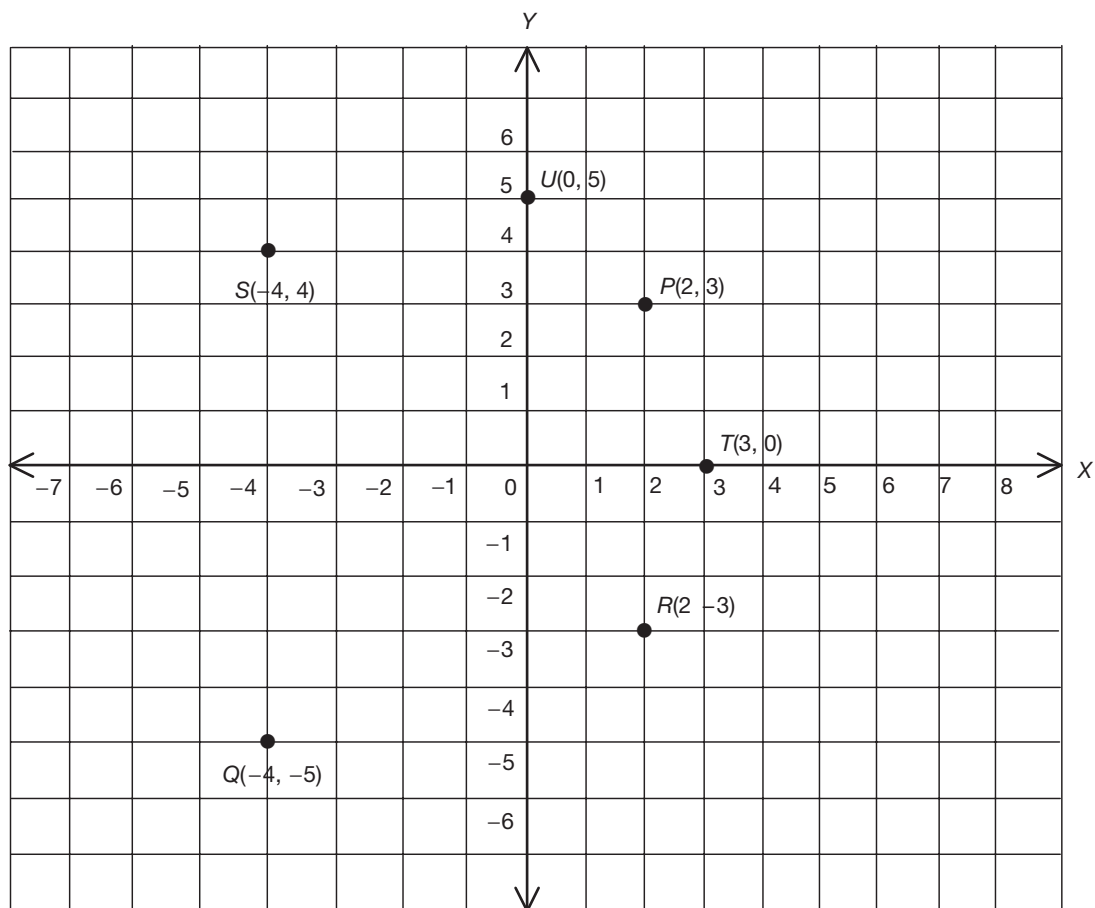


Figure 3.2

EXAMPLE 3.10

Plot the following points on the coordinate plane. What do you observe? (See Fig. 3.3)

(a) $(-2, 3)$, $(-1, 3)$, $(0, 3)$, $(1, 3)$, $(2, 3)$

(b) $(4, 2)$, $(4, 1)$, $(4, 0)$, $(4, -1)$, $(4, -2)$

SOLUTION

(a) $(-2, 3)$, $(-1, 3)$, $(0, 3)$, $(1, 3)$, $(2, 3)$

(i) The above points lie on the same straight line which is perpendicular to the Y-axis.

(ii) The y -coordinates of all the given points are the same, i.e., $y = 3$.

(iii) Hence, the straight line passing through the given points is represented by $y = 3$.

(iv) Therefore, the line $y = 3$ is parallel to X -axis which intersects Y -axis at $(0, 3)$.

(b) $(4, 2), (4, 1), (4, 0), (4, -1), (4, -2)$

(i) The above points lie on the same straight line which is perpendicular to the X -axis.

(ii) The x -coordinates of all the given points is the same, i.e., $x = 4$.

(iii) Hence, the straight line passing through the given points is represented by $x = 4$.

(iv) Therefore, the line $x = 4$ is parallel to the Y -axis which intersects the X -axis at $(4, 0)$.

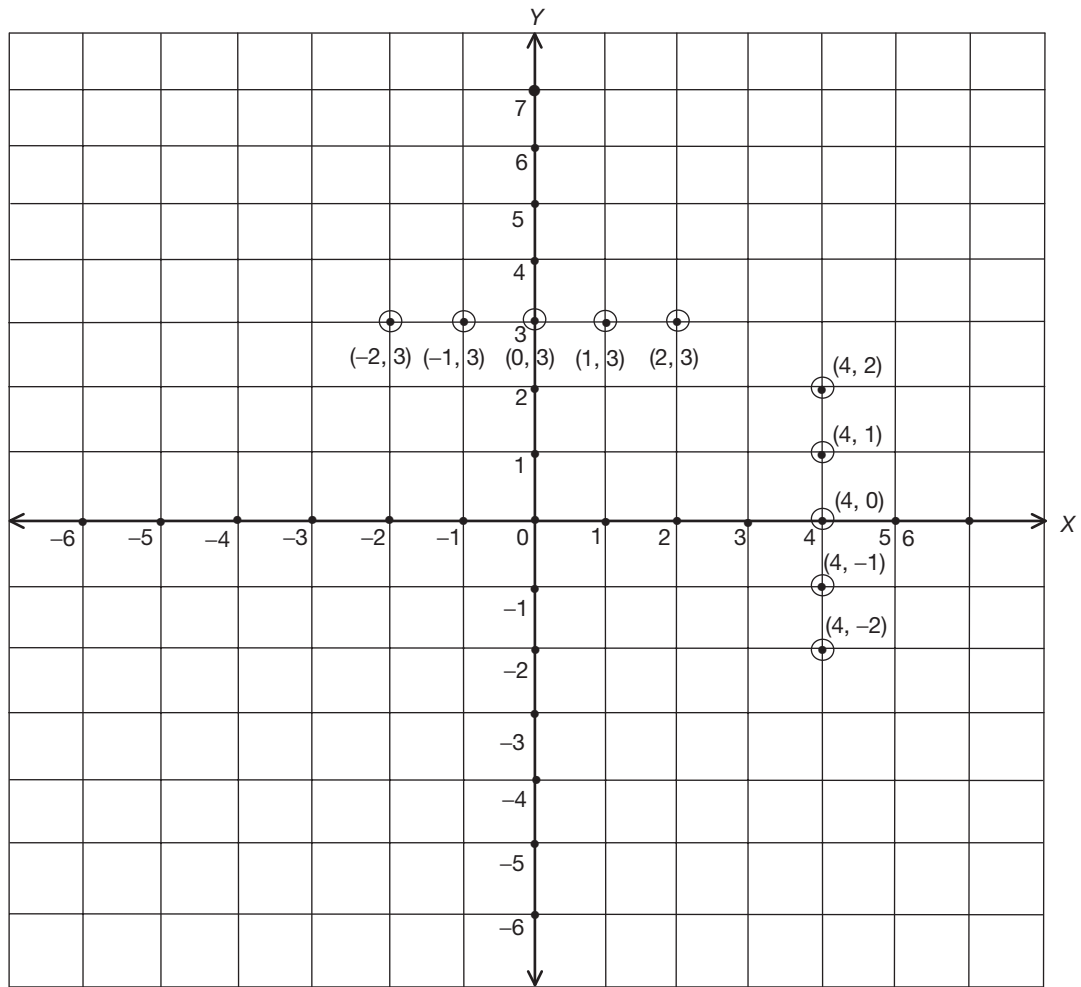


Figure 3.3

Notes

1. The y -coordinate of every point on the X -axis is zero, i.e., $y = 0$. Therefore the X -axis is denoted by $y = 0$.
2. The x -coordinate of every point on the Y -axis is zero, i.e., $x = 0$. Therefore the Y -axis is denoted by $x = 0$.

EXAMPLE 3.11

Plot the following points on the coordinate plane and what do you observe?

$(-3, 3)$, $(-2, 2)$, $(-1, 1)$, $(0, 0)$, $(1, -1)$, $(2, -2)$, $(3, -3)$

SOLUTION

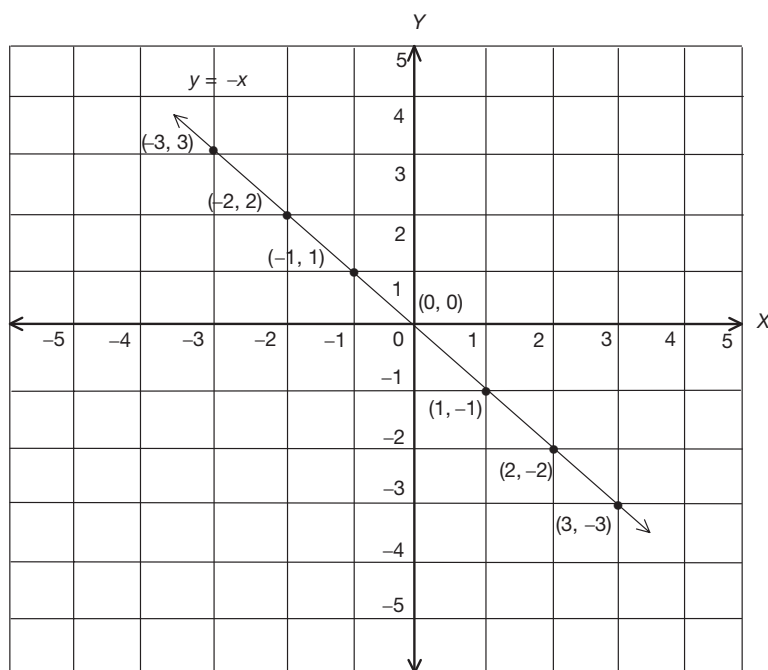


Figure 3.4

- (a) All the given points lie on the same straight line.
- (b) Every point on the straight line represents $y = -x$.
- (c) The above line with the given ordered pairs is represented by the equation $y = -x$.

EXAMPLE 3.12

Draw the graph of the equation $y = 3x$ where R is the replacement set for both x and y .

SOLUTION

x	-2	-1	0	1	2
$y = 3x$	-6	-3	0	3	6

Some of the ordered pairs which satisfy the equation $y = 3x$ are $(-1, -3)$, $(-2, -6)$, $(0, 0)$, $(1, 3)$, $(2, 6)$.

By plotting the above points on the graph sheet, we get the following.

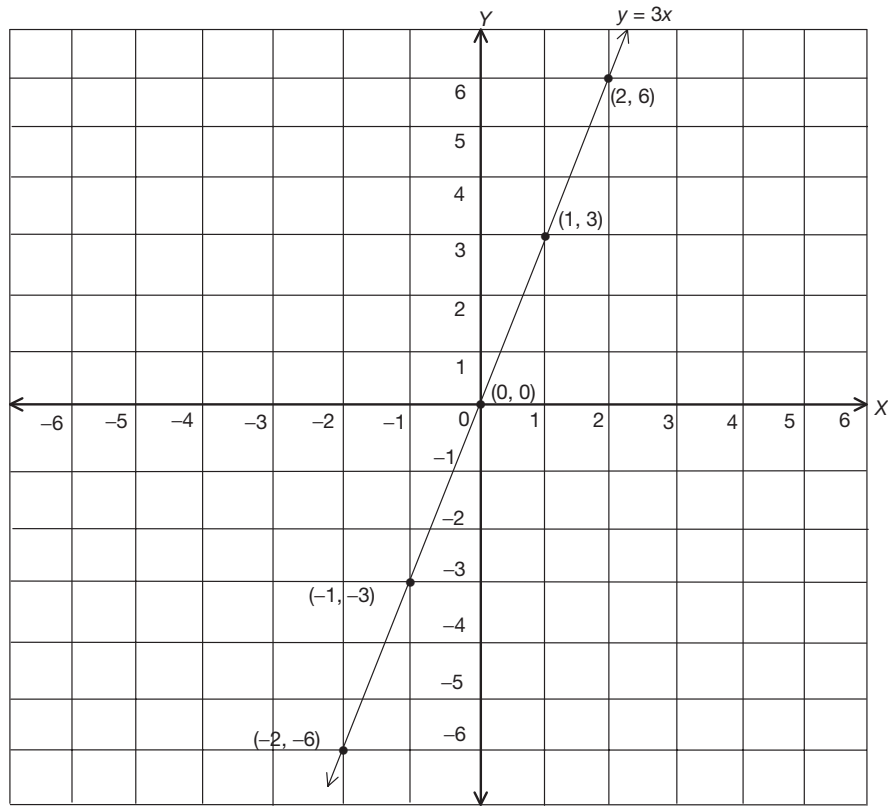


Figure 3.5

EXAMPLE 3.13

Draw the graph of the equations $x + y = -1$ and $x - y = 5$.

SOLUTION

(a) $x + y = -1$

x	-3	-2	-1	0	1	2	3
$y = -1 - x$	2	1	0	-1	-2	-3	-4

Some of the ordered pairs which satisfy the equation $x + y = -1$ are $(-3, 2)$, $(-2, 1)$, $(-1, 0)$, $(0, -1)$, $(1, -2)$, $(2, -3)$, $(3, -4)$.

(b) $x - y = 5$

x	-2	-1	0	1	2	3	4
$y = x - 5$	-7	-6	-5	-4	-3	-2	-1

$\therefore$ Some of the ordered pairs which satisfy the equation $x - y = 5$ are $(-2, -7)$, $(-1, -6)$, $(0, -5)$, $(1, -4)$, $(2, -3)$, $(3, -2)$, $(4, -1)$.

The ordered pairs which satisfy the equations $x + y = -1$ and $x - y = 5$ are plotted on a graph paper. We find that each equation represents a line.

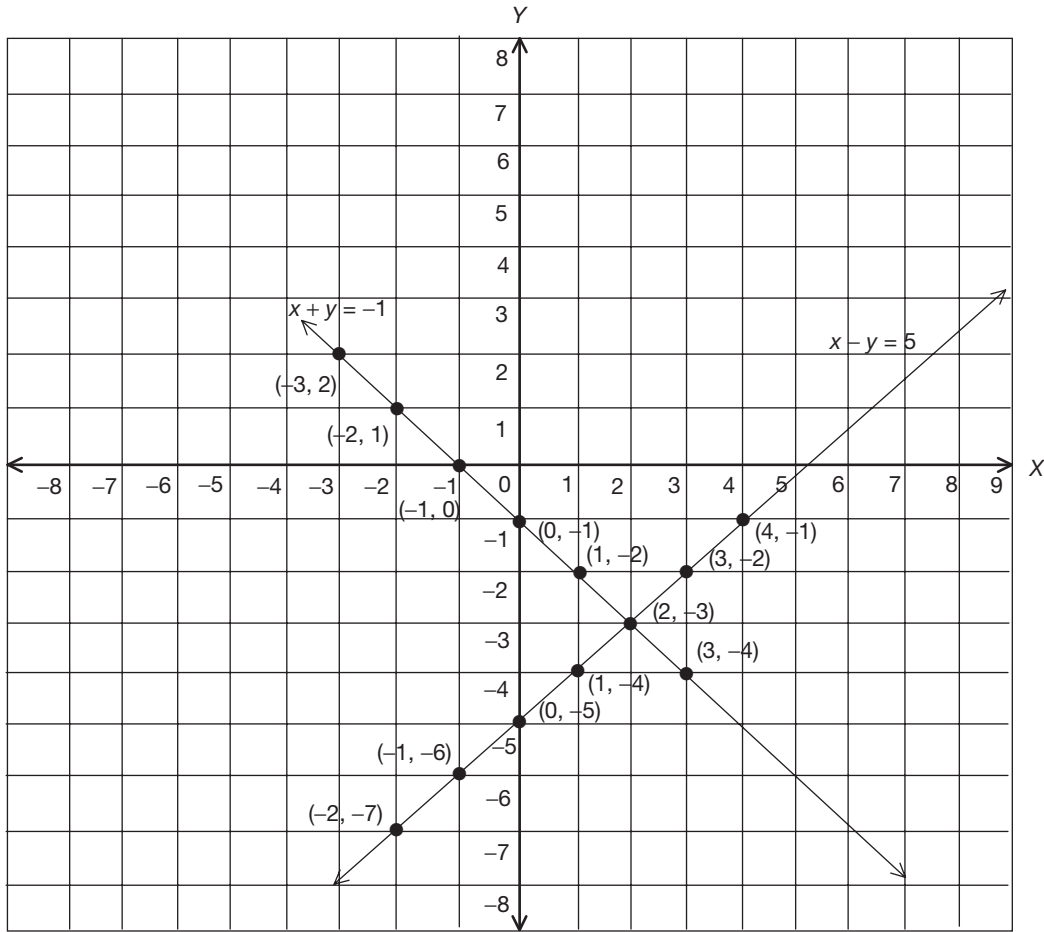


Figure 3.6

From the graph, we notice that the two given lines intersect at the point $(2, -3)$.

That is, lines $x + y = -1$ and $x - y = 5$ have a common point $(2, -3)$. Therefore, $(2, -3)$ is the solution of the equations $x + y = -1$ and $x - y = 5$.

Verification:

$$x + y = -1 \quad (1)$$

$$x - y = 5 \quad (2)$$

Solving Eqs. (1) and (2), we get,

$$x = 2 \text{ and } y = -3.$$

$\therefore (2, -3)$ is the solution of $x + y = -1$ and $x - y = 5$.

Note From the above example, we notice that we can find the solution for simultaneous equations by representing them in graphs, i.e., by using the graphical method.

Nature of Solutions

When we try to solve a pair of equations we could arrive at three possible results. They are, having

1. a unique solution.
2. an infinite number of solutions.
3. no solution.

Let the pair of equations be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where a_1 and a_2 are the coefficients of x ; b_1 and b_2 are the coefficients of y ; while c_1 and c_2 are the known constant quantities.

1. A pair of equations having a unique solution:

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of equations will have a unique solution.

We have solved such equations in the previous examples of this chapter.

2. A pair of equations having infinite solutions:

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have infinite number of solutions.

Note In fact this means that there are no two equations as such and one of the two equations is simply obtained by multiplying the other with a constant. These equations are known as dependent equations.

Example:

$$\begin{aligned} 3x + 4y &= 8 \\ 9x + 12y &= 24 \end{aligned}$$

For these two equations $a_1 = 3$, $a_2 = 9$, $b_1 = 4$, $b_2 = 12$, $c_1 = -8$, $c_2 = -24$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{Since, } \frac{3}{9} = \frac{4}{12} = \frac{-8}{-24}$$

The above pair of equations will have infinite solutions.

3. A pair of equations having no solution at all:

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have no solution.

Notes

1. In other words, the two equations will contradict each other or be inconsistent with each other.
2. A pair of equations is said to be consistent if it has a solution (finite or infinite).

Example:

$$\begin{aligned} 5x + 6y &= 30 \\ 10x + 12y &= 40 \end{aligned}$$

For these two equations, $a_1 = 5$, $a_2 = 10$, $b_1 = 6$, $b_2 = 12$, $c_1 = -30$, $c_2 = -40$.

Here,

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \\ \frac{5}{10} &= \frac{6}{12} \neq \frac{-30}{-40}. \end{aligned}$$

Hence, the pair of equations have no solution at all.

Word Problems and Application of Simultaneous Equations

In this chapter, we have discussed earlier that it is essential to have as many equations as there are unknown quantities to be determined. In word problems also, it is necessary to have as many independent conditions as there are unknown quantities to be determined.

Let us understand with the help of the following examples as to how word problems can be solved using simultaneous equations.

EXAMPLE 3.14

The sum of the successors of two numbers is 42 and the difference of their predecessors is 12. Find the numbers.

SOLUTION

Let the two numbers be x and y .

Given that,

$$\begin{aligned}(x + 1) + (y + 1) &= 42 \\ \Rightarrow x + y &= 40\end{aligned}\tag{1}$$

Also,

$$\begin{aligned}(x - 1) - (y - 1) &= 12 \\ \Rightarrow x - y &= 12\end{aligned}\tag{2}$$

Adding Eqs. (1) and (2), we get,

$$\begin{aligned}2x &= 52 \\ \Rightarrow x &= 26\end{aligned}$$

Substituting $x = 26$ in any of the Eqs. (1) and (2) we get $y = 14$.

$\therefore$ The two numbers are 26 and 14.

EXAMPLE 3.15

In a fraction, if numerator is increased by 2 and denominator is decreased by 3, then the fraction becomes 1. Instead, if numerator is decreased by 2 and denominator is increased by 3, then the fraction becomes $\frac{3}{8}$. Find the fraction.

SOLUTION

Let the fraction be $\frac{a}{b}$.

Applying the first condition, we get $\frac{a+2}{b-3} = 1$

$$\begin{aligned}\Rightarrow a + 2 &= b - 3 \\ \Rightarrow a - b &= -5\end{aligned}\tag{1}$$

Applying the second condition, we get $\frac{a-2}{b+3} = \frac{3}{8}$

$$\begin{aligned}\Rightarrow 8a - 16 &= 3b + 9 \\ \Rightarrow 8a - 3b &= 25\end{aligned}\tag{2}$$

Solving the Eqs. (1) and (2) using any of the methods discussed earlier, we get $a = 8$ and $b = 13$.

$\therefore$ The fraction is $\frac{8}{13}$.

EXAMPLE 3.16

In a box, the total number of ₹2 coins and ₹5 coins is 20. If the total coins amount to ₹76, find the number of coins of each denomination.

SOLUTION

Let the number of ₹2 coins and ₹5 coins be x and y respectively.

$$\text{Given, } x + y = 20\tag{1}$$

$$2x + 5y = 76\tag{2}$$

Solving Eqs. (1) and (2), we get

$$x = 8 \text{ and } y = 12.$$

That is, The number of ₹2 coins = 8 and number of ₹5 coins = 12.

EXAMPLE 3.17

Four years ago, the age of a person was thrice that of his son. Eight years later, the age of the person will be twice that of his son. Find the present ages of the person and his son.

SOLUTION

Let the present ages of the person and his son be x years and y years respectively.

Given,

$$\begin{aligned}x - 4 &= 3(y - 4) \\ \Rightarrow x - 4 &= 3y - 12 \\ \Rightarrow x - 3y &= -8\end{aligned}\tag{1}$$

Also,

$$\begin{aligned}x + 8 &= 2(y + 8) \\ \Rightarrow x + 8 &= 2y + 16 \\ \Rightarrow x - 2y &= 8\end{aligned}\tag{2}$$

Solving the Eqs. (1) and (2), we get

$$x = 40 \text{ and } y = 16.$$

EXAMPLE 3.18

For what value of k do the set of equations $4x - (3k + 2)y = 20$ and $(11k - 3)x - 10y = 40$ have infinite solutions?

SOLUTION

Given equations are

$$4x - (3k + 2)y = 20 \quad (1)$$

$$(11k - 3)x - 10y = 40 \quad (2)$$

System of Eqs. (1) and (2) have infinite solutions, if $\frac{4}{11k - 3} = \frac{-(3k + 2)}{-10} = \frac{-20}{-40}$

$$\Rightarrow \frac{4}{11k - 3} = \frac{1}{2}$$

$$\Rightarrow 8 = 11k - 3$$

$$\Rightarrow 11k = 11$$

$$\Rightarrow k = 1.$$

TEST YOUR CONCEPTS

Very Short answer Type Questions

- If $99x + 101y = 400$ and $101x + 99y = 600$, then $x + y$ is _____.
- The number of common solutions for the system of linear equations $5x + 4y + 6 = 0$ and $10x + 8y = 12$ is _____.
- Sum of the heights of A and B is 320 cm and the difference of heights of A and B is 20 cm. The height of B can be _____. (140 cm/145 cm/150 cm).
- If $a : b = 7 : 3$ and $a + b = 20$, then $b =$ _____.
- If $\frac{1}{x} + \frac{1}{y} = k$ and $\frac{1}{x} - \frac{1}{y} = k$, then the value of y is _____. (is 0/does not exist).
- If $p + q = k$, $p - q = n$ and $k > n$, then q is _____ (positive/negative).
- If $a + b = x$, $a - b = y$ and $x < y$, then b is _____ (positive/negative).
- If $3a + 2b + 4c = 26$ and $6b + 4a + 2c = 48$, then $a + b + c =$ _____.
- Sum of the ages of X and Y , 12 years ago, was 48 years and sum of the ages of X and Y , 12 years hence will be 96 years. Present age of X is _____.
- If the total cost of 3 chairs and 2 tables is ₹1200 and the total cost of 12 chairs and 8 tables is ₹4800, then the cost of each chair must be ₹200 and each table must be ₹300. (True/False)
- If the total cost of 2 apples and 3 mangoes is ₹22, then the cost of each apple and each mango must be ₹5 and ₹4 respectively, (where cost of each apple and mango is an integer). (True/False)
- Number of non-negative integral solutions for the equation $2x + 3y = 12$ is _____.
- Two distinct natural numbers are such that the sum of one number and twice the other number is 6. The two numbers are _____.
- If $2x + 3y = 5$ and $3x + 2y = 10$, then $x - y =$ _____.
- If $a + b = p$ and $ab = p$, then find the value of p . (where a and b are positive integers).

Short Answer Type Questions

- Solve: $331a + 247b = 746$ and $247a + 331b = 410$.
- Six gallery seats and three balcony seats for a play were sold for ₹162. Four gallery seats and five balcony seats were sold for ₹180. Find the price of a gallery seat and the price of a balcony seat.
- If the numerator of a fraction is increased by 2 and the denominator is decreased by 4, then it becomes 2. If the numerator is decreased by 1 and the denominator is increased by 2, then it becomes $\frac{1}{3}$. Find the fraction.
- Solve: $\frac{1}{x} + \frac{1}{y} = 6$, $\frac{1}{y} + \frac{1}{z} = 7$ and $\frac{1}{z} + \frac{1}{x} = 5$.
- Solve: $\frac{2}{x+y} - \frac{1}{x-y} = 11$ and $\frac{5}{x+y} + \frac{4}{x-y} = 8$.
- Jaydeep starts his job with a certain monthly salary and earns a fixed increment in his monthly salary at the middle of every year, starting from the first year. If his monthly salary was ₹78000 at the end of 6 years of service and ₹84000 at the end of 12 years of service, find his initial salary and annual increment.
- Alok was asked to find, $\frac{6}{7}$ of a number but instead he multiplied it by $\frac{7}{6}$. As a result he got an answer, which was more than the correct answer by 299. What was the number?
- Shriya has certain number of 25 paise and 50 paise coins in her purse. If the total number of coins is 35 and their total value is ₹15.50, find the number of coins of each denomination.
- For what value of k , will the following pair of linear equations have no solution?
 $2x + 3y = 1$ and $(3k - 1)x + (1 - 2k)y = 2k + 3$.
- Solve: $\frac{x}{a} + \frac{y}{b} = a^2 + b^2$ and $\frac{x}{a^2} + \frac{y}{b^2} = a + b$.



Essay Type Questions

26. Solve: $x - 2y + z = 0$, $9x - 8y + 3z = 0$ and $2x + 3y + 5z = 36$.
27. Four friends P, Q, R and S have some money. The amount with P equals the total amount with the others. The amount with Q equals one-third of the total amount with the others. The amount with R equals one-fifth of the total amount with the others. The amount with S equals one-eleventh of the total amount with the others. The sum of the smallest and the largest amounts with them is ₹210. Find the sum of the amounts with the other two (in ₹).
28. The population of a town is 25000. If in the next year the number of males were to increase by 5% and that of females by 3%, the population would grow to 26010. Find the number of males and females in the town at present.
29. What is the solution set of $\frac{12}{2x+3y} + \frac{5}{3x-2y} = -7$ and $\frac{8}{2x+3y} + \frac{6}{3x-2y} = -10$?
30. Tito purchased two varieties of ice cream cups, vanilla and strawberry – spending a total amount of ₹330. If each vanilla cup costs ₹25 and each strawberry cup costs ₹40, then in how many different combinations could he have purchased the ice cream cups?

CONCEPT APPLICATION

Level 1

1. For what value of k do the equations $3(k-1)x + 4y = 24$ and $15x + 20y = 8(k+13)$ have infinite solutions?
- (a) 1 (b) 4
(c) 3 (d) 2
2. If the system of equations $4x + py = 21$ and $px - 2y = 15$ has unique solution, then which of the following could be the value of p ?
- (A) 103 (B) 105
(C) 192 (D) 197
- (a) Both (A) and (B)
(b) Both (C) and (D)
(c) (A), (B) and (D)
(d) All of (A), (B), (C) and (D)
3. If the system of equations $2x - 3y = 3$ and $-4x + qy = \frac{p}{2}$ is inconsistent, which of the following cannot be the value of p ?
- (a) -24 (b) -18
(c) -12 (d) -36
4. The semi perimeter of a triangle exceeds each of its side by 5, 3 and 2 respectively. What is the perimeter of the triangle?
- (a) 12 (b) 10
(c) 15 (d) 20
5. If (p, p) is the solution of system of equations $ax + by + (t-s) = 0$ and $bx + ay + (s-r) = 0$, ($a \neq b$), then which of the following must be true?
- (a) $2r = s + t$ (b) $2t = r + s$
(c) $2s = r + t$ (d) $r + s + t = 0$
6. If $173x + 197y = 149$ and $197x + 173y = 221$, then find (x, y) .
- (a) (3, -2) (b) (2, 1)
(c) (1, -2) (d) (2, -1)
7. Mallesh has some cows and some hens in his shed. The total number of legs is 92 and the total number of heads is 29. Find the number of cows in his shed.
- (a) 12 (b) 14
(c) 17 (d) 19
8. Total cost of 14 pens and 21 books is ₹130 and the total cost of 6 pens and p books is ₹90. Which of the following cannot be the value of p ?



- (a) 8 (b) 9
(c) 10 (d) 11

9. If an ordered pair satisfying the equations $2x - 3y = 18$ and $4x - y = 16$ also satisfies the equation $5x - py - 23 = 0$, then find the value of p .

- (a) 1 (b) 2
(c) -1 (d) -2

10. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are such that a_1, b_1, c_1, a_2, b_2 and c_2 are consecutive integers in the same order, then find the values of x and y .

- (a) 1, -2 (b) 2, -3
(c) 1, 3 (d) Data insufficient

11. If we increase the length by 2 units and the breadth by 2 units, then the area of rectangle is increased by 54 square units. Find the perimeter of the rectangle (in units).

- (a) 44 (b) 50
(c) 56 (d) Cannot be determined

12. A mother said to her son, 'the sum of our present ages is twice my age 12 years ago and nine years hence, the sum of our ages will be thrice my age 14 years ago'. What is her son's present age? (in years)

- (a) 8 (b) 12
(c) 15 (d) 10

13. A told B, 'when I was as old as you are now, then your age was four years less than half of my present age'. If the sum of the present ages of A and B is 61 years, what is B's present age? (in years)

- (a) 9 (b) 25
(c) 43 (d) 36

14. If the system of equations $4x - 5y = 6$ and $-12x + ay = b$ is inconsistent, which of the following cannot be the value of b ?

- (a) -16 (b) -18
(c) -20 (d) -22

15. A sum of ₹400 was distributed among the students of a class. Each boy received ₹8 and each girl received ₹4. If each girl had received ₹10, then each boy would have received ₹5. Find the total number of students of the class.

- (a) 40 (b) 50
(c) 60 (d) 70

Level 2

16. Dheeraj has twice as many sisters as he has brothers. If Deepa, Dheeraj's sister has the same number of brothers as she has sisters, then Deepa has how many brothers?

- (a) 2 (b) 3
(c) 4 (d) Cannot be determined

17. The total cost of six books, five pencils and seven sharpeners is ₹115 and that of eight books, ten pencils and fourteen sharpeners is ₹190, then which of the following article's cost can be found uniquely?

- (a) Book (b) Pencil
(c) Sharpener (d) None of these

18. The sum of the speeds of a boat in still water and the speed the current is 10 kmph. If the boat takes 40% of the time to travel downstream when compared to that upstream, then find the difference of

the speeds of the boat when travelling upstream and down stream.

- (a) 3 kmph (b) 6 kmph
(c) 4 kmph (d) 5 kmph

19. In a fraction, if the numerator is decreased by 1 and the denominator is increased by 1, then the fraction becomes $\frac{1}{2}$. Instead, if the numerator is increased by 1 and the denominator is decreased by 1, then the fraction becomes $\frac{4}{5}$. Find the numerator of the fraction.

- (a) 5 (b) 6
(c) 7 (d) 8

20. Ram has 18 coins in the denominations of ₹1, ₹2 and ₹5. If their total value is ₹54 and the number of ₹2 coins are greater than that of ₹5 coins, then find the number of ₹1 coins with him.

- (a) 2 (b) 3
(c) 4 (d) Cannot be determined
21. If the ordered pair $(\sin \theta, \cos \theta)$ satisfies the system of equations $mx + ny + a + b = a - b$ and $nx + my + 2b = 0$, then find the value of θ where $0 \leq \theta \leq 90^\circ$. ($m \neq n$)
(a) 30° (b) 45°
(c) 50° (d) Cannot be determined
22. Swaroop can row 16 km downstream and 8 km upstream in 6 hours. He can row 6 km upstream and 24 km downstream in 6 hours. Find the speed of Swaroop in still water.
(a) 6 kmph (b) 8 kmph
(c) 3 kmph (d) 5 kmph
23. A two-digit number is formed by either subtracting 17 from nine times the sum of the digits or by adding 21 to 13 times the difference of the digits. Find the number.
(a) 37 (b) 73
(c) 71 (d) Cannot be determined
24. Swathi starts her job with certain monthly salary and earns a fixed increment every year. If her salary was ₹22500 per month after 6 years of service and ₹30000 per month after 11 years of service. Find her salary after 8 years of service (in ₹).
(a) 24000 (b) 25500
(c) 26000 (d) 24500
25. A three digit number abc is 459 more than the sum of its digits. What is the sum of the 2 digit number ab and the 1-digit number a ?
(a) 71 (b) 61
(c) 51 (d) Cannot be determined
26. The following sentences are the steps involved in solving the inequation $|5x + 4| > 5x - 4$. Arrange in sequential order from first to last.
(A) $10x < 0$
(B) $5x + 4 < -5x + 4$ or $5x + 4 > 5x - 4$
(C) $x < 0$
(D) $5x + 4 < -5x + 4$
(a) BDAC (b) DBAC
(c) ABDC (d) BADC
27. The following sentences are the steps involved in solving the inequation $5x + 2 > 7x - 4$. Arrange them in sequential order from first to last.
(A) Solution set = $\{0, 1, 2\}$ ($\because x \in W$)
(B) $-2x > -6$
(C) $5x - 7x > -4 - 2$
(D) $x < 3$
(a) CBAD (b) BCDA
(c) CBDA (d) BCAD
28. The total cost of 6 erasers and 9 pens is at least ₹102 and the cost of each eraser is at most ₹5. Find the minimum possible cost (in rupees) of a pen. The following are the steps involved in solving the above problem. Arrange them in sequential order.
(A) Let the cost of each eraser be ₹ x and cost of each pen be ₹ y .
(B) $6x + 9y \geq 102$ and $y \leq 5$.
(C) $6 \times 5 + 9y \geq 102 \Rightarrow 9y \geq 72 \Rightarrow y \geq 8$.
(D) The minimum possible cost of a pen is ₹8.
(a) ABDC (b) ABCD
(c) DABC (d) ACBD

Level 3

29. An examination consists of 100 questions. Two marks are awarded for every correct option. If one mark is deducted for every wrong option and half mark is deducted for every question left, then a person scores 135. Instead, if half mark is deducted for every wrong option and one mark is deducted for every question left, then the person scores 133. Find the number of questions left unattempted by the person.
(a) 14 (b) 16
(c) 10 (d) 12
30. Ram, Shyam, Tarun and Varun together had a total amount of ₹240 with them. Ram had half



of the total amount with the others. Shyam had one-third of the total amount with the others. Tarun had one-fourth of the total amount with the others. Find the amount with Varun (in ₹).

- (a) 64 (b) 70
(c) 52 (d) 58

31. Ramu had 13 notes in the denominations of ₹10, ₹50 and ₹100. The total value of the notes with him was ₹830. He had more of ₹100 notes than that of ₹50 notes with him. Find the number of ₹10 notes with him.

- (a) 4 (b) 3
(c) 2 (d) 5

32. A, B, C and D share a certain amount amongst themselves. B sees that the other three get 3 times what he himself gets. C sees that the other three get 4 times what he gets, while D sees that the other three get 5 times what he gets. If the sum of the largest and smallest shares is 99, what is the sum of the other two shares?

- (a) 99 (b) 81
(c) 64 (d) 54

33. If $3|x| + 5|y| = 8$ and $7|x| - 3|y| = 48$, then find the value of $x + y$.

- (a) 5 (b) -4
(c) 4 (d) The value does not exist

34. The cost of 2 puffs, 14 cups of coffee and 5 pizzas is ₹356. The cost of 20 puffs, 7 cups of coffee and 15 pizzas is ₹830. Find the cost of 38 puffs and 25 pizzas. (in ₹)

- (a) 1296 (b) 1104
(c) 1304 (d) Cannot be determined

35. A father's present age is 6 times his son's present age. Thirty years hence the father's age will be ten years less than twice the son's age. After how many years will the son's age be half of the father's present age?

- (a) 20 (b) 30
(c) 10 (d) 15

36. Ramesh had a total of 30 coins in his purse of denominations ₹5 and ₹2. If the total amount with him is ₹120, then find the number of ₹5 coins.

- (a) 20 (b) 10
(c) 4 (d) 12

37. Mukesh has some goats and hens in his shed. Upon counting, Mukesh found that the total number of legs is 112 and the total number of heads is 40. Find the number of hens in his shed.

- (a) 18 (b) 20
(c) 22 (d) 24

38. If the length and breadth of a room are increased by 1 m each, its area would increase by 31 m^2 . If the length is increased by 1 m and breadth is decreased by 1 m, the area would decrease by 9 m^2 . Find the area of the floor of the room, in m^2 .

- (a) 200 (b) 209
(c) 250 (d) 199

39. A hybrid mango tree, whose life span is 10 years, starts giving fruits from the first year onwards. In the n th year it produces $11n$ raw mangoes. But during the first half of the tree's life, every year, a certain number, which is constant, fail to ripen into fruits. During the second half of the tree's life, every year, the number of raw fruits that fail to ripen is half the corresponding number in the first half of the tree's life. In the fourth year of the tree's life, it produces 36 ripe mangoes. How many mangoes ripen during the 9th year of the tree's life?

- (a) 100 (b) 96
(c) 95 (d) 86

40. A teacher wanted to distribute 900 chocolates among the students of a class. Each boy received 12 chocolates and each girl received 6 chocolates. If each girl had been given 10 chocolates, then each boy would have received 5 chocolates. Find the number of students of the class.

- (a) 80 (b) 90
(c) 100 (d) 110

41. Sridevi purchased cakes of two varieties of soap, Lux and Dove—spending a total ₹360. If each Lux costs ₹30 and each Dove costs ₹40, then in how many different combinations could she have purchased the cakes?

- (a) 3 (b) 4
(c) 5 (d) 2

42. Venu has as many sisters as he has brothers. If Karuna, Venu's sister has thrice as many brothers as she has sisters, then Venu has how many sisters?
- (a) 1 (b) 2
(c) 3 (d) 4
43. The cost of two pencils, five erasers and eight sharpeners is ₹47. The cost of three pencils, three erasers and seven sharpeners is ₹42. Find the cost of twelve pencils, three erasers and eighteen sharpeners. (in ₹)
- (a) 37 (b) 92
(c) 138 (d) 111
44. Total cost of 15 erasers and 25 pencils is ₹185 and the total cost of 9 erasers and x pencils is ₹106. Which of the following cannot be the value of x ?
- (a) 12 (b) 10
(c) 13 (d) 15
45. In a fraction, if the numerator is decreased by 1 and the denominator is increased by 1, then the resulting fraction is $\frac{1}{4}$. Instead, if the numerator is increased by 1 and the denominator is decreased by 1, then the resulting fraction is $\frac{2}{3}$. Find the difference of the numerator and the denominator of the fraction.
- (a) 2 (b) 3
(c) 4 (d) 5
46. A two-digit number is such that, it exceeds the sum of the number formed by reversing the digits and sum of the digits by 4. Also, the original number exceeds the reversed number by 18. Find the product of the digits.
- (a) 48 (b) 36
(c) 42 (d) 56
47. Bhanu has a total of 40 coins of denominations 30 paise and 10 paise. The total amount with him is ₹9. Find the number of 10 paise coins with him.
- (a) 25 (b) 35
(c) 15 (d) 20
48. A father's present age is seven years less than 30 times of what his son's age was 20 years ago. Also, the father's present age is 31 years more than his son's present age. Find the sum of their present ages, in years.
- (a) 75 (b) 74
(c) 73 (d) 72
49. A and B, have some coins. If A gives 100 coins to B, then B will have twice the number of coins left with A. Instead, if B gives 40 coins to A, then A will have thrice the number of coins left with B. How many more coins does A have than B?
- (a) 64 (b) 88
(c) 75 (d) 96
50. Snehal can row 28 km downstream and 12 km upstream in 5 hours. He can row 21 km downstream and 10 km upstream in 4 hours. Find the speed of Snehal in still water.
- (a) 9 kmph (b) 8 kmph
(c) 6 kmph (d) 5 kmph
51. The ratio of monthly incomes of Mr X and Mr Y is 3 : 4 and the ratio of their monthly expenditures is 5 : 7. If the ratio of their monthly savings is 3 : 2 and Mr X saves ₹500 more than Mr Y per month, then find the monthly income of Mr Y.
- (a) ₹35000 (b) ₹32000
(c) ₹26000 (d) ₹22000
52. A two-digit number is seven times the sum of its digits. The number formed by reversing the digits is 6 more than half of the original number. Find the difference of the digits of the given number.
- (a) 2 (b) 3
(c) 4 (d) 5
53. Sanjana travels 660 km, partly by train and partly by car. If she covers 300 km by train and the rest by car, it takes 13.5 hours. But, if she travels 360 km by train and the rest by car, she takes 30 minutes longer. Find the time taken by Sanjana if she travels 660 km by car. (in hours)
- (a) 13 (b) 14
(c) 12 (d) 11
54. In a test of 50 questions, each correct answer fetches two marks and each wrong answer fetches $-\frac{1}{2}$ marks. A candidate attempted all the questions



and scored 40 mark. How many questions did he attempt correctly?

- (a) 24 (b) 26
(c) 22 (d) 20

55. The average weight of the students of a class is 60 kg. If eight new students of average weight 64 kg

join the class, the average weight of the entire class becomes 62 kg. How many students were there in the class initially?

- (a) 12 (b) 10
(c) 8 (d) 14



TEST YOUR CONCEPTS

Very Short Answer Type Questions

- | | |
|--------------------|-------------------------|
| 1. 5 | 9. Cannot be determined |
| 2. zero | 10. False |
| 3. 150 cm | 11. False |
| 4. 6 | 12. 3 |
| 5. Does not exist. | 13. 4 and 1 |
| 6. Positive | 14. 5 |
| 7. Negative | 15. 4 |
| 8. 10 | |

Short Answer Type Questions

- | | |
|---|---|
| 16. $a = 3$ and $b = -1$ | 21. ₹72000, ₹1000 |
| 17. ₹15, ₹24 | 22. 966 |
| 18. $\frac{4}{7}$ | 23. Number of 25 paise coins is 8 and number of 50 paise coins is 27. |
| 19. $x = \frac{1}{2}, y = \frac{1}{4}$ | 24. $\frac{5}{13}$ |
| 20. $x = \frac{-1}{24}, y = \frac{7}{24}$ | 25. $x = a^3, y = b^3$ |

Essay Type Questions

- | | |
|---------------------------|-----------------------------------|
| 26. $x = 1, y = 3, z = 5$ | 29. $\left(\frac{1}{2}, 1\right)$ |
| 27. $x = 4$ and $y = 5$ | 30. 2 |
| 28. 13000, 12000 | |

CONCEPT APPLICATION

Level 1

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|--------|--------|--------|--------|---------|
| 1. (d) | 2. (d) | 3. (c) | 4. (d) | 5. (c) | 6. (d) | 7. (c) | 8. (b) | 9. (b) | 10. (a) |
| 11. (b) | 12. (b) | 13. (b) | 14. (b) | 15. (c) | | | | | |

Level 2

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 16. (b) | 17. (a) | 18. (b) | 19. (c) | 20. (b) | 21. (b) | 22. (d) | 23. (b) | 24. (b) | 25. (c) |
| 26. (a) | 27. (c) | 28. (b) | | | | | | | |

Level 2

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 29. (a) | 30. (c) | 31. (b) | 32. (b) | 33. (d) | 34. (c) | 35. (c) | 36. (a) | 37. (d) | 38. (b) |
| 39. (c) | 40. (d) | 41. (d) | 42. (b) | 43. (d) | 44. (d) | 45. (c) | 46. (a) | 47. (c) | 48. (a) |
| 49. (b) | 50. (a) | 51. (d) | 52. (c) | 53. (d) | 54. (b) | 55. (c) | | | |



CONCEPT APPLICATION

Level 1

- Condition for infinite solutions is, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- If two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are inconsistent, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
- (i) Semi perimeter (s) = $\frac{a+b+c}{2}$.
(ii) $a + 5 = s$; $b + 3 = s$; $c + 2 = s$.
- If $x = y$, then $a_1 = a_2$, $b_1 = b_2$ and $c_1 = c_2$.
- Add two equations.
- Each cow has 4 legs and each hen has 2 legs.
- Frame the linear equation and write $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Solve the first two equations and substitute (x, y) in the third equation.
- Take any consecutive integers for a_1 , b_1 , c_1 , a_2 , b_2 and c_2 and solve.

Or,

Substitute any convenient consecutive integers for a_1 , b_1 , c_1 , a_2 , b_2 and c_2 and solve.

- Frame linear equation by taking length and breadth as l and b respectively.
- Frame linear equations and solve.
- (i) Let the present ages of A and B be x years and y years respectively.
(ii) $y - (x - y) = \frac{x}{2} - 4$.
(iii) $x + y = 61$.
- If the system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
- Let the cost of each book and each pencil and each sharpener be b , p and s respectively. Then frame the equations.

Level 2

- (i) Let the number of sons and daughters that Dheeraj's parents have be b and d respectively.
(ii) Form the equations and solve them.
- Let the cost of each book and each pencil and each sharpener be b , p and s respectively. Then frame the equations.
- Let the fraction be $\frac{n}{a}$ and frame the linear equations.
- (i) Let the number of ₹1, ₹2 and ₹5 coins be x , y and z respectively.
(ii) From the information given, obtain an equation in y and z and proceed.
- If (p, q) is a solution of $ax + by + c = 0$ and $bx + ay + c = 0$, then $p = q$.
- (i) $10x + y = 9(x + y) - 17$.
(ii) $10x + y = 13(x - y) + 21$.
(iii) Solve the above equations.
- Frame linear equations and solve.
- (i) Let the three digit number be $100x + 10y + z$.
(ii) Frame the equations and solve them.
- BDAC are in sequential order from first to last.
- CBDA are in sequential order from first to last.
- ABCD is the required sequential order.

Level 3

- (i) $x + y + z = 100$, $2x - y - \frac{1}{2}z = 135$; $2x - \frac{y}{2} - z = 133$
where x , y , and z are the number of correct

answers, wrong answers and unattempted questions respectively.

- (ii) Solve the above equations.

30. (i) Let r, s, t, v , be the amounts, with Ram, Shyam, Tarun, Varun respectively.

$$(ii) r = \frac{1}{2}(s + t + v), s = \frac{1}{3}(r + t + v),$$

$$t = \frac{1}{4}(r + s + v), v + s + t + v = 240.$$

(iii) Solve the above equation set to get v .

32. Let the total amounts be $4B$ or $5C$ or $6D$ and $B + D = 99$.

33. (i) Let $|x| = a$ and $|y| = b$ and solve linear equations which are in a and b .

(ii) Let $|x| = k_1$ and $|y| = k_2$ and solve them.

(iii) $|x| = \pm x, |y| = \pm y$.

34. Let the cost of each puff be p .

Let the cost of each coffee be c .

Let the cost of each pizza be z .

$$\text{Then } 2p + 14c + 5z = 356 \quad (1)$$

$$\text{Also, } 20p + 7c + 15z = 830 \quad (2)$$

Multiply Eq. (2) by 2,

$$\text{we have } 40p + 14c + 30z = 1660 \quad (3)$$

subtracting Eq. (1) from Eq. (3),

$$\text{i.e., } 38p + 25z = 1304.$$

35. Let the present ages of father and son be f and s respectively.

$$\text{Then, } f = 6s \quad (1)$$

The second condition gives

$$f + 30 = 2(s + 30) - 10 \Rightarrow f = 2s + 20$$

$$\therefore 6s - 2s = 20 \quad (\because \text{from Eq. (1)})$$

$$\Rightarrow s = 5 \text{ and } f = 30$$

Half the father's present age is 15. After 10 years, the son's age will be 15.

36. Let the number of 5-rupee coins be x .

Let the number of 2-rupee coins be y .

$$\therefore x + y = 30 \quad (1)$$

$$\text{Also } 5x + 2y = 120 \quad (2)$$

Multiplying Eq. (1) by 2

$$2x + 2y = 60 \quad (3)$$

subtracting Eq. (3) from Eq. (2),

$$3x = 60$$

$$x = 20$$

37. Let the number of goats and hens, be x and y respectively.

$$\text{Given, } 4x + 2y = 112 \quad (1)$$

$$x + y = 40 \quad (2)$$

Solving Eqs. (1) and (2), we get

$$x = 16 \text{ and } y = 24$$

$$\therefore \text{Number of hens} = 24$$

38. Let the length of the room = 1 m

Breadth of the room = b m

Area of the room = lb m².

The length and breadth are increased by 1 m each and the area increases by 31 m²

$$(l + 1)(b + 1) = lb + 31$$

$$\Rightarrow lb + l + b + 1 = lb + 31$$

$$\Rightarrow l + b = 30 \quad (1)$$

If length increased by 1 m and breadth decreased by 1 m, then the area decreases by

$$9 \text{ m}^2, \text{ i.e., } (l + 1)(b - 1) = lb - 9$$

$$\Rightarrow lb - l + b - 1 = lb - 9$$

$$\Rightarrow l - b = 8 \quad (2)$$

On solving Eq. (1) and Eq. (2), we get $l = 19, b = 11$

$$\Rightarrow \text{Area of the floor} = lb = 19 \times 11$$

$$= 209 \text{ m}^2$$

39. Every year in the first half of the tree's life, the number of mangoes that cannot become ripen fruit is y (say).

The number of fruits produced in the x th year = $11x - y$

Number of fruits produced in 4th year = 36

$$\Rightarrow 11(4) - y = 36 \Rightarrow y = 8$$

Number of fruits produced in x th year of the

$$\text{second half life} = 11x - \frac{y}{2}$$

Number of fruits produced in the 9th year

$$= 11(9) - \frac{8}{2} = 99 - 4 = 95.$$



40. Let there be x boys and y girls in the class.

Case 1:

$$12x + 6y = 900$$

$$2x + y = 150 \quad (1)$$

Case 2:

$$5x + 10y = 900$$

$$x + 2y = 180 \quad (2)$$

Solving (1) and (2), we get $x = 40$ and $y = 70$

$\therefore$ Number of students in the class $= 70 + 40 = 110$.

41. Let the number of Lux cakes be x and Dove cakes be y .

$$\text{Given } 30x + 40y = 360$$

$$\Rightarrow 3x + 4y = 36$$

$$\Rightarrow x = \frac{36 - 4y}{3}$$

$$\Rightarrow x = 12 - \frac{4y}{3}$$

As x and y are positive integers, $y = 3$ or 6

$$\Rightarrow x = 8, 1$$

There are two combinations.

- 42.

No. of Girls	No. of Boys
x	$(x + 1)$ (including Venu)
$(x - 1)$ (without Karuna)	$3(x - 1)$ (including Venu)

$$\Rightarrow 3(x - 1) = x + 1$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

43. Let the cost of each pencils be P .

Let the cost of each sharpner be s .

Let the cost of each eraser be e .

$$\text{Then, } 2p + 5e + 8s = 47 \quad (1)$$

$$3p + 3e + 7s = 42 \quad (2)$$

Multiply Eq. (2) by 2 and subtract Eq. (1) from Eq. (2), we have

$$4p + e + 6s = 37 \quad (3)$$

Multiply Eq. (3) by 3

$$12p + 3e + 18s = 111$$

44. Let the cost of each eraser and each pencil be ₹ e and ₹ p respectively.

$$\Rightarrow 15e + 25p = 185 \text{ and } 9e + xp = 106$$

As the cost of these articles is unique,

$$\frac{15}{9} \neq \frac{25}{x}$$

$$\Rightarrow x \neq 15.$$

45. Let the fraction be $\frac{x}{y}$.

Case 1:

$$\frac{x-1}{x+1} = \frac{1}{4} \Rightarrow 4x - 4 = y + 1$$

$$\Rightarrow 4x - y = 5 \quad (1)$$

Case 2:

$$\frac{x+1}{y-1} = \frac{2}{3}$$

$$\Rightarrow 3x + 3 = 2y - 2$$

$$\Rightarrow 3x - 2y + 5 = 0 \quad (2)$$

Solving Eqs. (1) and (2), we get $x = 3$ and $y = 7$

$\therefore$ The required difference is 4.

46. Let x, y be the tens and units digits respectively.

Sum of the digits $= x + y$

Value of the number $= 10x + y$

$$10x + y = x + y + 4 + 10y + x$$

$$8x - 10y = 4 \text{ or } 4x - 5y = 2 \quad (1)$$

Also given that

$$10x + y = 10y + x + 18$$

$$x - y = 2 \quad (2)$$

Multiply Eq. (2) by 4, we have

$$4x - 4y = 8 \quad (3)$$

Subtract Eq. (1) from Eq. (3)

$$y = 6$$

Substitute $y = 6$ in (2), we have $x = 8$

The product of the digits $= 48$

47. Let the total number of 30 paise coins with him = x .

Let the total number of 10 paise coins with him = y .

$$\text{Then } x + y = 40$$

$$\text{Also, } 30x + 10y = 900$$

Solving for x and y we have $y = 15$.

48. Let the present age of father be f years.

Let son's present age be s years.

$$\text{Then } f = 30(s - 20) - 7 \quad (1)$$

$$f = s + 31 \quad (2)$$

$$s + 31 = 30s - 600 - 7$$

$$638 = 29s$$

$$22 = s$$

$$\Rightarrow f = 53$$

Sum of their ages = 75 years.

49. Let A and B have x and y coins respectively.

Case 1:

If A gives 100 coins to B.

$$y + 100 = 2(x - 100)$$

$$2x - y = 300 \quad (1)$$

Case 2:

If B gives 40 coins to A

$$x + 40 = 3(y - 40)$$

$$\Rightarrow x - 3y + 160 = 0 \quad (2)$$

Solving Eqs. (1) and (2), we get

$$x = 212 \text{ and } y = 124$$

$\therefore$ The required difference = $212 - 124 = 88$.

50. Let Snehal's speed and the speed of the stream be x kmph and y kmph respectively.

$$\text{Given, } \frac{28}{x + y} + \frac{12}{x - y} = 5 \quad (1)$$

$$\text{and, } \frac{21}{x + y} + \frac{10}{x - y} = 4 \quad (2)$$

$$\text{Let } \frac{1}{x + y} = a \text{ and } \frac{1}{x - y} = b, \text{ then}$$

$$(1) \Rightarrow 28a + 12b = 5 \quad (3)$$

$$\text{and } (2) \Rightarrow 21a + 10b = 4 \quad (4)$$

Solving Eqs. (3) and (4), we get

$$a = \frac{1}{14} \text{ and } b = \frac{1}{4}$$

$$\Rightarrow x + y = 14 \text{ and } x - y = 4$$

$$\Rightarrow x = 9 \text{ and } y = 5$$

$\therefore$ Snehal's speed in still water = 9 kmph

51. Let the monthly income of X and Y be $3x$, $4x$ and monthly expenditures be $5y$, $7y$

$$\text{Savings of X} = 3x - 5y$$

$$\text{Savings of Y} = 4x - 7y$$

$$\text{Given, } \frac{3x - 5y}{4x - 7y} = \frac{3}{2} \text{ and}$$

$$(3x - 5y) - (4x - 7y) = 500$$

$$\Rightarrow 6x - 10y = 12x - 21y$$

$$\Rightarrow 6x - 11y = 0 \quad (1)$$

$$\text{and } -x + 2y = 500 \quad (2)$$

Solving Eqs. (1) and (2), we get $y = 3000$ and $x = 5500$

$\therefore$ Monthly income of Mr Y = ₹ $4x$ = ₹22000.

52. Let the two-digit number be $10x + y$

$$\text{Given, } 10x + y = 7(x + y)$$

$$\Rightarrow 3x = 6y \Rightarrow x = 2y \quad (1)$$

$$\text{Also, } 10y + x = \frac{1}{2}(10x + y) + 6$$

$$\Rightarrow -8x + 19y = 12$$

$$\Rightarrow -16y + 19y = 12 (\because \text{from Eq. (1)})$$

$$\Rightarrow y = 4 \text{ and } x = 8 \therefore x - y = 4.$$

53. Let the speed of the train and car be t km/h and c km/h respectively.

$$\therefore \frac{300}{t} + \frac{360}{c} = 13.5 \quad (1)$$

$$\frac{360}{t} + \frac{300}{c} = 14 \quad (2)$$

On solving Eqs. (1) and (2), we get

$$t = 40 \text{ km/h and } c = 60 \text{ km/h}$$

The time taken by car to travel 660 km

$$= \frac{660}{60} = 11 \text{ hours.}$$





54. Let the number of questions which are correct be c and the questions which are wrong be w .

$$c + w = 50 \quad (1)$$

$$2c - \frac{w}{2} = 40$$

$$4c - w = 80 \quad (2)$$

$$\text{From Eqs. (1) and (2)} \Rightarrow 5c = 130 \Rightarrow c = 26$$

55. Let there be n students in the class. The total weight of the students after the eight new students join the class.

$$60n + 64(8) = 62(n + 8)$$

$$\Rightarrow 60n + 64(8) = 62n + 62(8)$$

$$\Rightarrow 62n - 60n = 64(8) - 62(8)$$

$$\Rightarrow 2n = 8(2) \Rightarrow n = 8$$

$\therefore$ Initially, there were 8 students in the class.