Shortcuts and Important Results to Remember

1 $(r+1)$th term from end in the expansion of $(x+y)^n = (r+1)$th term from beginning in the expansion of $(y+x)^n$.

2 If ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in AP, then $(n-2r)^2 = n+2$ or $r = \frac{1}{2}(n \pm \sqrt{(n+2)})$ for $r=2$, $n=7$ and for $r=5, n=7, 14$.

3 Four consecutive binomial coefficients can never be in AP.

4 Three consecutive binomial coefficients can never be in GP or HP.

5 If a, b, c, d are four consecutive coefficients in the expansion of $(1+x)^n$, then $\frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in AP.

(i) $\frac{a}{a+b} + \frac{c}{c+d} = 2\left(\frac{b}{b+c}\right)$

(ii) $\left(\frac{b}{b+c}\right)^2 > \frac{ac}{(a+b)(c+d)}$

6 If greatest term in $(1+x)^{2n}$ has the greatest coefficient, then $\frac{n}{n+1} < x < \frac{n+1}{n}$.

7 (a) The coefficient of x^{n-1} in the expansion of

$$(x-1)(x-2)(x-3)\dots(x-n) = -(1+2+3+\dots+n) = -\frac{n(n+1)}{2} = -{}^{n+1}C_2$$

(b) The coefficient of x^{n-1} in the expansion of

$$(x+1)(x+2)(x+3)\dots(x+n) = (1+2+3+\dots+n) = \frac{n(n+1)}{2} = {}^{n+1}C_2$$

8 The number of terms in the expansion of

$$(x+a)^n + (x-a)^n = \begin{cases} \frac{n+2}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

9 The number of terms in the expansion of

$$(x+a)^n - (x-a)^n = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

10 The number of terms in the expansion of multinomial $(x_1 + x_2 + x_3 + \dots + x_m)^n$, when $x_1, x_2, x_3, \dots, x_m \in C$ and $n \in N$, is ${}^{n+m-1}C_{m-1}$.

11 The number of terms in the expansion of $\left(ax^p + \frac{b}{x^p} + c\right)^n$, where $n, p \in N$ and a, b, c are constants, is $2n+1$.

12 If the coefficients of pth and qth terms in the expansion of $(1+x)^n$ are equal, then $p+q = n+2$, where $p, q, n \in N$.

13 If the coefficients of x^r, x^{r+1} in the expansion of $\left(a + \frac{x}{b}\right)^n$ are equal, then $n = (r+1)(ab+1) - 1$, where $n, r \in N$ and a, b are constants.

14 Coefficient of x^m in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ = Coefficient of T_{r+1}, where $r = \frac{np-m}{p+q}$, where $p, q, n \in N$ and a, b are constants.

15 The term independent of x in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ is T_{r+1}, where $r = \frac{np}{p+q}$, where $n, p, q \in N$ and a, b are constants.

16 Sum of the coefficients in the expansion of $(ax+by)^n$ is $(a+b)^n$, where $n \in N$ and a, b are constants.

17 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$ and $p+q=1$, then

(i) $\sum_{r=0}^{n} r \cdot C_r \cdot p^r \cdot q^{n-r} = np$

(ii) $\sum_{r=0}^{n} r^2 \cdot C_r \cdot p^r \cdot q^{n-r} = n^2p^2 + npq$

18 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then

(i) $\sum_{r=0}^{n} r \cdot C_r = n \cdot 2^{n-1}$ (ii) $\sum_{r=0}^{n} \frac{C_r}{r+1} = \frac{2^{n+1}-1}{n+1}$

(iii) $\sum_{r=0}^{n} r^2 \cdot C_r = n(n+1)2^{n-2}$ (iv) $\sum_{r=0}^{n} (-1)^r \cdot r \cdot C_r = 0$

(v) $\sum_{r=0}^{n} \frac{(-1)^r C_r}{r+1} = \frac{1}{n+1}$

(vi) $\sum_{r=0}^{n} (-1)^r \frac{C_r}{r} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

(vii) $\sum_{r=0}^{n} (-1)^r \cdot r^2 \cdot C_r = 0$

(viii) $\sum_{r=0}^{n} (-1)^r \cdot (a-r)(b-r)C_r = 0, \forall n > 3$

(ix) $\sum_{r=0}^{n} (-1)^r (a-r)(b-r)(c-r)C_r = 0, \forall n > 3$

(x) $\sum_{r=0}^{n} (-1)^r (a-r)^3 C_r = 0, \forall n > 3$

(xi) $\sum_{r=0}^{n} r(r-1)(r-2)\dots(r-k+1)C_r x^{r-k} = \frac{d^k}{dx^k}(1+x)^n$

for $k=2$, $\sum_{r=0}^{n} r(r-1)C_r = \frac{d^2}{dx^2}[(1+x)^n]_{x=1} = n(n-1)2^{n-2}$

and for $k=3$; $\sum_{r=0}^{n} r(r-1)(r-2)(-1)^{r-3}C_r$

$$= \frac{d^3}{dx^3}[(1+x)^n]_{x=-1} = 0$$